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Condensation of a saturated turbulent vapor jet onto a concurrent subcooled turbulent liquid jet

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CONDENSATION OF A SATURATED TURBULENT VAPOR JET ONTO A COCURRENT SUBCOOLED TURBULENT LIQUID JET

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CONDENSATION OF A SATURATED TURBULENT VAPOR JET ONTO A COCURRENT SUBCOOLED TURBULENT LIQUID JET - Heinz Jaster

I. ABSTRACT

An analytical study of the interaction region between turbulent jets of a subcooled liquid and its saturated vapor is presented. The geometry is plane; the jets are co-flowing. The shear stress is treated according to Prandtl's old theory of turbulence.

The solution predicts:

a) the boundary layer thickness of the liquid and of the vapor

b) the velocity distribution in the interaction region

c) the Stanton Number

d) the interfacial friction factor

as functions of density ratio, velocity ratio and $C_p \Delta T_s / h_{fg}$ only.

Numerical solutions are given in tabular and graphical form.

This thesis is accepted and approved in partial fulfillment of the requirements for the degree of Master of Science.

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I. ABSTRACT

An analytical study of the interaction region between turbulent jets of a subcooled liquid and its saturated vapor is presented. The geometry is plane; the jets are co-flowing. The shear stress is treated according to Prandtl's old theory of turbulence.

The solution predicts:

a) the boundary layer thickness of the liquid and of the vapor

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II. INTRODUCTION

Shown in Figure 1 is a simple sketch of the system under consideration.

A thin wall initially separates two parallel streams of turbulent fluids. The upper stream is a saturated vapor with free stream velocity u_v; the lower stream is the subcooled liquid of the same fluid with free stream temperature T_ℓ and free stream velocity u_ℓ. As the streams come into contact, the vapor condenses onto the liquid due to differences in temperature and velocity. The rate of heat transfer is to be determined.

The regions above ϕ_v and below ϕ_ℓ represent uniform flow regimes for the vapor and its liquid respectively. The line ϕ_i defines the interface between the phases. The regions between ϕ_i and ϕ_ℓ and between ϕ_v and ϕ_i depict the boundary layer zones of the liquid and its vapor respectively.

Many practical devices exist in which a turbulent vapor stream is in contact with a subcooled turbulent liquid jet and subsequently condenses onto the surface of the liquid. Due to the turbulent motion of the fluids, it is possible to attain condensation rates which greatly surpass even those reported for film condensation processes. These very large condensation rates make this configuration desirable for use in direct contact condensers, condensing two-phase jet pumps, and other heat transfer devices.

III. ANALYSIS

1. The Boundary Layer Momentum Equation

The analysis is to be restricted to velocities low enough to make vapor compressibility effects negligible. Furthermore, only the case $u_v > u_\ell$ will be treated. The analysis can easily be extended to the case $u_v/u_\ell < 1$, a subject which will be further explored in the discussion on Prandtl's theory of turbulent shear, and in Appendix E.

The x-momentum boundary layer equation for steady, incompressible, isobaric, two-dimensional flow can be stated as

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial \tau_{xy}}{\partial y} , \tag{1}$$

where u and v are the time averaged x and y-components of velocity respectively, ρ is the fluid density and τ_{xy}, the shear stress acting on the fluid in the x-direction.

Prandtl gives the relationship

$$\tau_{xy} = \rho L^2 \frac{\partial u}{\partial y} \left| \frac{\partial u}{\partial y} \right| \tag{2}$$

where L is the turbulent mixing length.

Abramovich [1] conducted studies which showed that constant velocity lines in submerged jets are straight. That is

$$u/u_o = g(\tfrac{y}{x}) \tag{3}$$

where u_o is the velocity of the undisturbed jet and g is some function.

To account for the similarity of the velocity distribution, the mixing length must vary linearly with distance, that is

$$L = bx \tag{4}$$

where b is a constant. Therefore,

$$\frac{\partial \tau}{\partial y} = \pm \frac{\partial}{\partial y} [\rho b^2 x^2 \frac{\partial^2 u}{\partial y^2}] = \pm 2\rho b^2 x^2 \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2} \tag{5}$$

where the positive sign applies to positive velocity gradients and the negative sign to negative velocity gradients. For the coordinate system chosen in Figure 1 and for $u_v > u_\ell$, the positive sign is to be used.

To simplify further calculations let

$$a^3 \equiv 2b^2 . \tag{6}$$

If $\phi \equiv y/ax$ is chosen as an alternate coordinate for y, then

$$u/u_o = r(\phi) . \tag{7}$$

Defining a function $F(\phi)$ by

$$\frac{dF(\phi)}{d\phi} = r(\phi) \tag{8}$$

and defining the stream function ψ by

$$u \equiv \frac{\partial\psi}{\partial y}, \quad v \equiv -\frac{\partial\psi}{\partial x}\ , \tag{9}$$

the following expressions are obtained:

$$\psi = u_o a\ x\ F(\phi) \tag{10}$$

$$u = u_o F'(\phi) \tag{11}$$

$$v = u_o a(\phi F'(\phi) - F(\phi))\ . \tag{12}$$

Substituting equations (4), (5), (6), (11) and (12) into (1) and transforming coordinates, the momentum boundary layer equation becomes

$$F'''(\phi) + F(\phi) = 0\ . \tag{13}$$

The solution to equation (13) is

$$F(\phi) = C_1 e^{-\phi} + e^{\phi/2}\left(C_2 \cos\frac{\sqrt{3}}{2}\phi + C_3 \sin\frac{\sqrt{3}}{2}\phi\right)\ . \tag{14}$$

Various investigators have made measurements to determine the constant a for submerged jets. Abramovich [1], comparing his analytical work to experimental data by Albertson et al [2], gives a = 0.09. Kuethe [5] gives a = 0.0875, Goertler [4] found a = 0.098. The fluid for all of the above experiments was air.

We will, for purposes of this thesis, assume a = 0.09.

2. Boundary Conditions and Compatibility Equations

The system under consideration has two flow regimes. Let the liquid velocity be defined by $u = u_\ell F'(\phi)$, and the vapor velocity by $u = u_v G'(\phi)$, where $G(\phi)$ is an analogous function to $F(\phi)$. We obtain:

$$u = u_\ell F' \tag{15}$$

$$v = u_\ell a(\phi F' - F) \tag{16}$$

$$F = C_1 e^{-\phi} + e^{\phi/2}\left(C_2 \cos \frac{\sqrt{3}}{2}\phi + C_3 \sin \frac{\sqrt{3}}{2}\phi\right) \tag{17}$$

$$u = u_v G' \tag{18}$$

$$v = u_v a(\phi G' - G) \tag{19}$$

$$G = A_1 e^{-\phi} + e^{\phi/2}\left(A_2 \cos \frac{\sqrt{3}}{2}\phi + A_3 \sin \frac{\sqrt{3}}{2}\phi\right) \tag{20}$$

where the function F is for the liquid region and G for the vapor region.

To solve for the constants of integration and the magnitudes of ϕ_v, ϕ_i and ϕ_ℓ, nine conditions must be imposed.

Along ϕ_ℓ, the x-component of velocity is equal to the free stream velocity:

$$u(\phi_\ell) = u_\ell F'(\phi_\ell) = u_\ell$$

$$F'(\phi_\ell) = 1 \ . \tag{21}$$

The y-component of velocity equals zero at the edge of the liquid boundary layer:

$$v(\phi_\ell) = u_\ell a(\phi_\ell F'(\phi_\ell) - F(\phi_\ell)) = 0$$

$$F(\phi_\ell) = \phi_\ell \ . \tag{22}$$

The velocity gradient equals zero along ϕ_ℓ:

$$\left.\frac{\partial u}{\partial y}\right|_{\phi_\ell} = \frac{u_\ell}{ax} F''(\phi_\ell) = 0$$

$$F''(\phi_\ell) = 0 \ . \tag{23}$$

At the edge of the vapor boundary layer the velocity is equal to the vapor free stream velocity:

$$u(\phi_v) = u_v G'(\phi_v) = u_v \ .$$

$$G'(\phi_v) = 1 \ . \tag{24}$$

The velocity gradient is zero along ϕ_v:

$$\left.\frac{\partial u}{\partial y}\right|_{\phi_v} = \frac{u_v}{ax} G''(\phi_v) = 0$$

$$G''(\phi_v) = 0 \; . \tag{25}$$

Let us now develop the matching conditions at the interface (ϕ_i).

Considering conservation of mass for the control volume indicated in Figure 2 we obtain

$$\dot{m}x = + \int_{y_\ell}^{y_i} \rho_\ell u dy - \int_{y_\ell}^{o} u_\ell \rho_\ell dy$$

and (26)

$$\dot{m}x = - \int_{y_i}^{y_v} \rho_v u dy + \int_{o}^{y_v} \rho_v u_v dy - \int_{o}^{x} v \rho_v dx$$

where $\dot{m}$ is the rate of mass condensation per unit area, it is assumed to be constant with x.

After a coordinate transformation, the above equations reduce to:

$$\dot{m}x = ax\rho_\ell u_\ell [\int_{\phi_\ell}^{\phi_i} F'(\phi)d\phi + \phi_\ell]$$

and

$$\dot{m}x = ax\rho_v u_v [\int_{\phi_v}^{\phi_i} G'(\phi)d\phi + \phi_v - (\phi_v G'(\phi_v) - G(\phi_v))]$$

or:

$$\dot{m} = a\rho_\ell u_\ell F(\phi_i) \tag{27}$$

$$\dot{m} = a\rho_v u_v G(\phi_i) \tag{28}$$

with the aid of equations (22) and (24). Therefore:

$$G(\phi_i) = \frac{\rho_\ell u_\ell}{\rho_v u_v} F(\phi_i) \ . \tag{29}$$

Consider an energy balance around the control volume indicated in Figure 3, neglecting the effects of changes in potential and kinetic energies. Let:

h_f = enthalpy of the liquid = $h_o + C_p(T-T_o)$

h_o = enthalpy at reference temperature T_o

h_v = enthalpy of the saturated vapor

= $h_{fg} + h_o + C_p(T_v-T_o)$

T_v = temperature of the vapor

h_{fg} = latent heat

C_p = constant pressure specific heat of the liquid

The energy balance is:

$$\int_0^{y_v} u_v\rho_v h_v dy + \int_{y_\ell}^{0} u_\ell \rho_\ell h_f dy$$

$$= \int_0^{x} v_v\rho_v h_v dx + \int_{y_\ell}^{y_i} u\rho_\ell h_f dy$$

$$+ \int_{y_i}^{y_v} u\rho_v h_v dy \; . \tag{30}$$

To solve the second integral on the right side of equation (30), we need to know the temperature distribution within the liquid boundary layer.

Let us assume that the temperature varies linearly with ϕ and furthermore, that the momentum boundary layer has the same thickness as the temperature boundary layer.

The first of these assumptions is a fair approximation to measurements made by Abramovich [1] and by Reichardt [11]. The second assumption seems reasonable for u_v/u_ℓ large, since turbulent mixing should dominate the conduction mechanism. However, for u_v/u_ℓ near one, the momentum boundary layer thickness tends toward zero, while a finite temperature boundary layer will be sustained by conduction. So, if we restrict the application of the analysis to $u_v/u_\ell > 1$, we can write:

$$\frac{T-T_\ell}{T_i-T_\ell} = \frac{\phi-\phi_\ell}{\phi_i-\phi_\ell} \tag{31}$$

where T_i is the temperature of the liquid at the interface.

Making the necessary y-ϕ transformations and applying equations (13), (22), (23), (24) and (29) we arrive at

$$F''(\phi_i) = (\phi_i - \phi_\ell) \frac{\dfrac{h_{fg}}{C_p(T_v - T_\ell)} + \dfrac{T_v - T_i}{T_v - T_\ell}}{1 - \dfrac{T_v - T_i}{T_v - T_\ell}} F(\phi_i) \quad . \tag{32}$$

To exploit the last conservation law at our disposal, consider the control volume indicated in Figure 4 for purposes of making a momentum balance. The conservation equation is

$$\int_0^{y_v} \rho_v u_v^2 dy + \int_{y_\ell}^{0} \rho_\ell u_\ell^2 dy = \int_0^x \rho_v v_v u_v dx + \int_{y_i}^{y_v} \rho_v u^2 dy$$
$$+ \int_{y_\ell}^{y_i} \rho_\ell u^2 dy \quad . \tag{33}$$

By equation (13),

$$F'^2 = \frac{d}{d\phi}(FF') + \frac{1}{2}\frac{d}{d\phi}(F'')^2 \quad . \tag{34}$$

Using this result as well as equations (21), (23), (24), (25) and (29), equation (33) can be rewritten as:

$$\frac{1}{2} G''^2(\phi_i) + \frac{u_\ell}{u_v}\frac{\rho_\ell}{\rho_v} F(\phi_i)G'(\phi_i)$$

$$+ \frac{\rho_\ell}{\rho_v}\frac{u_\ell^2}{u_v^2} \left(F(\phi_i)F'(\phi_i) + \frac{1}{2} F''^2(\phi_i)\right) = 0 \,. \qquad (35)$$

One more condition is to be imposed to make the system determinate.

At the interface (ϕ_i), the tangential shear stress on the vapor side is equal to the shear stress on the liquid side.

If the angle between the interface and the x-axis is small, the above condition can be formalized as:

$$\rho_v L^2 \left(\frac{\partial^2 u}{\partial y^2}\right)^2_{\phi_i,\ \text{vapor}} = \rho_\ell L^2 \left(\frac{\partial^2 u}{\partial y^2}\right)^2_{\phi_i,\ \text{liquid}}$$

or:

$$[G''(\phi_i)]^2 = \frac{\rho_\ell u_\ell^2}{\rho_v u_v^2} [F''(\phi_i)]^2 \,. \qquad (36)$$

Substituting (35) into (37), the momentum condition becomes:

$$G'(\phi_i) = \frac{u_\ell}{u_v} F'(\phi_i) \qquad (37)$$

which is equivalent to the statement:

$$u(\phi_i)_{\text{vapor}} = u(\phi_i)_{\text{liquid}} \,.$$

For the sake of completeness let us discuss alternate boundary conditions. The possibility of setting higher order derivatives of u equal to zero along the liquid boundary layer edge was explored. The condition

$$\left.\frac{d^2u}{dy^2}\right|_{\phi_\ell} = 0$$

results in $F'''(\phi_\ell) = 0$, which implies (by equation (13)) that $F(\phi_\ell) = 0$. But if $F(\phi_\ell) = 0$ then, by equation (10), ϕ_ℓ becomes a streamline - an impossible condition. Setting

$$\left.\frac{d^3u}{dy^3}\right|_{\phi_\ell} = 0$$

results in $F''''(\phi_\ell) = 0$ which, again by equation (13), implies $F'(\phi_\ell) = 0$. But $u = u_\ell F'(\phi)$, i.e., $u_\ell = 0$.

Still higher derivatives will result in repetitions of the above inconsistencies.

The system can now be solved: nine equations exist to determine the nine unknowns. For the sake of convenience, the conditions will here be restated:

$$F'(\phi_\ell) = 1 \tag{21}$$

$$F(\phi_\ell) = \phi_\ell \tag{22}$$

$$F''(\phi_\ell) = 0 \tag{23}$$

$$G'(\phi_v) = 1 \tag{24}$$

$$G''(\phi_v) = 0 \tag{25}$$

$$G(\phi_i) = \frac{\rho_\ell}{\rho_v} \cdot \frac{u_\ell}{u_v} F(\phi_i) \tag{29}$$

$$F''(\phi_i) = F(\phi_i) \frac{\dfrac{h_{fg}}{C_p(T_v - T_\ell)} + \dfrac{T_v - T_i}{T_v - T_\ell}}{1 - \dfrac{T_v - T_i}{T_v - T_\ell}} (\phi_i - \phi_\ell) \tag{32}$$

$$[G''(\phi_i)]^2 = \frac{\rho_\ell}{\rho_v} \left(\frac{u_\ell}{u_v}\right)^2 [F''(\phi_i)]^2 \tag{36}$$

$$G'(\phi_i) = \frac{u_\ell}{u_v} F'(\phi_i) \quad . \tag{37}$$

3. Solution

Conditions (21), (22) and (23) will be used to solve for the constants of integration of equation (17). The application of equations (24), (25) and (29) will give us the constants of equation (20). Let:

$$\begin{aligned}
\alpha &= \phi - \phi_\ell \quad , \quad & \Delta\phi_\ell &= \phi_i - \phi_\ell \\
\beta &= \phi_v - \phi \quad , \quad & \Delta\phi_v &= \phi_v - \phi_i \\
\gamma &= \phi - \phi_i \quad , \quad & d &= \frac{\sqrt{3}}{2}\left(e^{\Delta\phi_v} + 2e^{-\Delta\phi_v/2} \cos \frac{\sqrt{3}}{2}\Delta\phi_v\right) .
\end{aligned} \tag{38}$$

Then, after some trigonometric and algebraic manipulations, we reach

$$F(\phi) = \frac{\phi_\ell - 1}{3} e^{-\alpha} + \frac{e^{\alpha/2}}{3}\left[(2\phi_\ell+1)\cos \frac{\sqrt{3}}{2}\alpha + \sqrt{3} \sin \frac{\sqrt{3}}{2}\alpha\right] \tag{39}$$

$$F'(\phi) = \frac{1-\phi_\ell}{3} e^{-\alpha} + \frac{e^{\alpha/2}}{3}\left[(\phi_\ell+2)\cos \frac{\sqrt{3}}{2}\alpha - \phi_\ell \sqrt{3} \sin \frac{\sqrt{3}}{2}\alpha\right] \tag{40}$$

$$F''(\phi) = \frac{\phi_\ell - 1}{3} e^{-\alpha} + \frac{e^{\alpha/2}}{3}\left[(1-\phi_\ell)\cos \frac{\sqrt{3}}{2}\alpha - \sqrt{3} (\phi_\ell+1)\sin \frac{\sqrt{3}}{2}\alpha\right] \tag{41}$$

$$G(\phi) = \frac{1}{2d}\left[\sqrt{3}\,\frac{\rho_\ell}{\rho_v}\,\frac{u_\ell}{u_v}\,F(\phi_i)\left(e^{\beta} + 2e^{-\beta/2}\cos\frac{\sqrt{3}}{2}\beta\right)\right.$$

$$+\, e^{(\beta-\gamma)/2}\left[\sin\frac{\sqrt{3}}{2}(\beta+\gamma) - \sqrt{3}\cos\frac{\sqrt{3}}{2}(\beta+\gamma)\right]$$

$$+\, 2e^{-(2\beta+\gamma)/2}\sin\frac{\sqrt{3}}{2}\gamma + e^{(\beta+2\gamma)/2}\left(\sqrt{3}\cos\frac{\sqrt{3}}{2}\beta\right.$$

$$\left.\left. -\, \sin\frac{\sqrt{3}}{2}\beta\right)\right] \tag{42}$$

$$G'(\phi) = \frac{1}{2d}\left[\sqrt{3}\,\frac{\rho_\ell}{\rho_v}\,\frac{u_\ell}{u_v}\,F(\phi_i)\left(-e^{\beta} + e^{-\beta/2}\left(\cos\frac{\sqrt{3}}{2}\beta\right.\right.\right.$$

$$\left.\left. +\, \sqrt{3}\sin\frac{\sqrt{3}}{2}\beta\right)\right) + e^{(\beta-\gamma)/2}\left[-\sin\frac{\sqrt{3}}{2}(\beta+\gamma)\right.$$

$$\left. +\, \sqrt{3}\cos\frac{\sqrt{3}}{2}(\beta+\gamma)\right] + e^{-(2\beta+\gamma)/2}\left(\sqrt{3}\cos\frac{\sqrt{3}}{2}\gamma\right.$$

$$\left. +\, \sin\frac{\sqrt{3}}{2}\gamma\right) + e^{(2\gamma+\beta)/2}\left(\sqrt{3}\cos\frac{\sqrt{3}}{2}\beta\right.$$

$$\left.\left. +\, \sin\frac{\sqrt{3}}{2}\beta\right)\right] \tag{43}$$

$$G''(\phi) = \frac{1}{2d}\left[\sqrt{3}\,\frac{\rho_\ell}{\rho_v}\,\frac{u_\ell}{u_v}\,F(\phi_i)\left(e^{\beta} + e^{-\beta/2}\left(-\cos\frac{\sqrt{3}}{2}\beta\right.\right.\right.$$

$$\left.\left. +\, \sqrt{3}\sin\frac{\sqrt{3}}{2}\beta\right)\right) + e^{(\beta-\gamma)/2}\left[-\sqrt{3}\cos\frac{\sqrt{3}}{2}(\beta+\gamma)\right.$$

$$\left. +\, \sin\frac{\sqrt{3}}{2}(\beta+\gamma)\right] + e^{-(2\beta+\gamma)/2}\left(\sqrt{3}\cos\frac{\sqrt{3}}{2}\gamma\right.$$

$$\left.\left. -\, \sin\frac{\sqrt{3}}{2}\gamma\right) + 2e^{(2\gamma+\beta)/2}\sin\frac{\sqrt{3}}{2}\beta\right]. \tag{44}$$

At $\phi = \phi_i$ these expressions reduce to:

$$F(\phi_i) = \frac{\phi_\ell - 1}{3} e^{-\Delta\phi_\ell} + \frac{e}{3}^{\Delta\phi_\ell/2} [(2\phi_\ell + 1)\cos \frac{\sqrt{3}}{2} \Delta\phi_\ell$$

$$+ \sqrt{3} \sin \frac{\sqrt{3}}{2} \Delta\phi_\ell] \tag{45}$$

$$F'(\phi_i) = \frac{1 - \phi_\ell}{3} e^{-\Delta\phi_\ell} + \frac{e}{3}^{\Delta\phi_\ell/2} [(\phi_\ell + 2)\cos \frac{\sqrt{3}}{2} \Delta\phi_\ell$$

$$- \sqrt{3}\, \phi_\ell \sin \frac{\sqrt{3}}{2} \Delta\phi_\ell] \tag{46}$$

$$F''(\phi_i) = \frac{\phi_\ell - 1}{3} e^{-\Delta\phi_\ell} + \frac{e}{3}^{\Delta\phi_\ell/2} [(1 - \phi_\ell)\cos \frac{\sqrt{3}}{2} \Delta\phi_\ell$$

$$- \sqrt{3}\, (\phi_\ell + 1)\sin \frac{\sqrt{3}}{2} \Delta\phi_\ell] \tag{47}$$

$$G(\phi_i) = \frac{\rho_\ell}{\rho_v} \frac{u_\ell}{u_v} F(\phi_i) \tag{48}$$

$$G'(\phi_i) = \frac{\sqrt{3}}{2d} [\frac{\rho_\ell}{\rho_v} \frac{u_\ell}{u_v} F(\phi_i)(-e^{\Delta\phi_v} + e^{-\Delta\phi_v/2}(\cos \frac{\sqrt{3}}{2} \Delta\phi_v$$

$$+ \sqrt{3} \sin \frac{\sqrt{3}}{2} \Delta\phi_v)) + e^{-\Delta\phi_v}$$

$$+ 2e^{\Delta\phi_v/2} \cos \frac{\sqrt{3}}{2} \Delta\phi_v] \tag{49}$$

$$G''(\phi_i) = \frac{\sqrt{3}}{2d} [\frac{\rho_\ell}{\rho_v} \frac{u_\ell}{u_v} (e^{\Delta\phi_v} + e^{-\Delta\phi_v/2}(\sqrt{3} \sin \frac{\sqrt{3}}{2} \Delta\phi_v$$

$$- \cos \frac{\sqrt{3}}{2} \Delta\phi_v)) F(\phi_i) + e^{\Delta\phi_v/2}(\sqrt{3} \sin \frac{\sqrt{3}}{2} \Delta\phi_v$$

$$- \cos \frac{\sqrt{3}}{2} \Delta\phi_v) + e^{-\Delta\phi_v}] \,. \tag{50}$$

Equations (45) and (47) permit us to solve equation (32) for ϕ_ℓ in terms of $\Delta\phi_\ell$ and $\frac{T_v - T_i}{T_v - T_\ell}$. The ratio $\frac{T_v - T_i}{T_v - T_\ell}$ ranges, depending on fluid properties and flow parameters, from zero to one. To make the presentation of numerical solutions wieldy, let us proceed with the assumption that $\frac{T_v - T_i}{T_v - T_\ell}$ is negligible as far as equation (32) is concerned. This hypothesis is discussed further in Appendix D. We now have an explicit solution for ϕ_ℓ:

$$\phi_\ell =$$

$$\frac{e^{-\Delta\phi_\ell}(1-\Delta\phi_\ell w)+e^{\Delta\phi_\ell/2}\left[\sqrt{3}(1+\Delta\phi_\ell w)\sin\frac{\sqrt{3}}{2}\Delta\phi_\ell+(\Delta\phi_\ell w-1)\cos\frac{\sqrt{3}}{2}\Delta\phi_\ell\right]}{e^{-\Delta\phi_\ell}(1-\Delta\phi_\ell w)-e^{\Delta\phi_\ell/2}\left[(1+2\Delta\phi_\ell w)\cos\frac{\sqrt{3}}{2}\Delta\phi_\ell+\sqrt{3}\sin\frac{\sqrt{3}}{2}\Delta\phi_\ell\right]} \tag{51}$$

where $w = \frac{h_{fg}}{C_p \Delta T_s}$. The sole unused equations are (36) and (37). They are made to yield to computer computations to solve for $\Delta\phi_\ell$ and $\Delta\phi_v$.

If we define h the convection heat transfer coefficient by:

$$h = \frac{q}{A}\frac{1}{\Delta T_s} \tag{52}$$

where

$\frac{q}{A}$ $\equiv$ heat transfer per unit area from the vapor to the liquid,

$\Delta T_s \equiv T_v - T_\ell$,

then, since

$$\frac{q}{A} = \dot{m}h_{fg}, \tag{53}$$

we arrive at

$$h = \frac{a\rho_\ell u_\ell h_{fg} F(\phi_i)}{\Delta T_s} \; . \tag{54}$$

To nondimensionalize the heat transfer coefficient, let us introduce the liquid Stanton Number:

$$S \equiv \frac{h}{\rho_\ell u_\ell C_p} = \frac{a h_{fg} F(\phi_i)}{C_p \Delta T_s} \; . \tag{55}$$

The following are some useful expressions which can be easily derived from the foregoing development.

The shear stress in the liquid region:

$$\tau = \frac{\rho_\ell a u_\ell^2}{2} [F''(\phi)]^2 \; . \tag{56}$$

The shear stress in the vapor region:

$$\tau = \frac{\rho_v a u_v^2}{2} [G''(\phi)]^2 \; . \tag{57}$$

The dimensionless shear stress in the liquid region:

$$\frac{\tau}{\tau_i} = [\frac{F''(\phi)}{F''(\phi_i)}]^2 \; . \tag{58}$$

The dimensionless shear stress in the vapor region:

$$\frac{\tau}{\tau_i} = \frac{\rho_v}{\rho_\ell} (\frac{u_v}{u_\ell})^2 [\frac{G''(\phi)}{F''(\phi_i)}]^2 \; . \tag{59}$$

The interfacial friction factor:

$$f \equiv \frac{2\tau_i}{\rho_v u_v^2} = a \frac{\rho_\ell}{\rho_v} (\frac{u_\ell}{u_v})^2 [F''(\phi_i)]^2 \tag{60}$$

or

$$f = \frac{\rho_\ell}{\rho_v} (\frac{u_\ell}{u_v})^2 (\Delta\phi_\ell)^2 S^2 \; . \tag{61}$$

The dimensionless condensation rate:

$$\frac{\dot{m}}{\rho_\ell u_\ell} = aF(\phi_i) \; . \tag{62}$$

IV. RESULTS

1. Tables of Numerical Results

The following tables are solutions for the system under discussion. Each table fixes $C_p \Delta T_s / h_{fg}$ as well as the density ratio. Results for the Stanton Numbers and interfacial friction factors, as well as for some fluid dynamic variables, are listed as functions of the velocity ratio. The range of the physical parameters was chosen to be useful for a water system. They should, however, serve well for a variety of fluids.

The computations were performed on a digital computer. The numerical method employed is discussed in Appendix C.

TABULATION OF SOLUTIONS

$C_p\ T_s/h_{fg}$ = .010

ρ_ℓ/ρ_v = 1000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.016	.099	-.365	.365	.854	.002	1.016	.066
3	.021	.117	-.467	.464	1.024	.002	1.034	.108
4	.025	.119	-.543	.535	1.124	.003	1.052	.146
5	.027	.117	-.605	.593	1.192	.003	1.071	.181
6	.030	.115	-.660	.642	1.243	.003	1.091	.214
7	.032	.112	-.710	.685	1.283	.004	1.112	.247
8	.035	.109	-.756	.724	1.315	.004	1.134	.278
9	.037	.106	-.799	.759	1.342	.004	1.156	.309
10	.039	.104	-.840	.791	1.364	.004	1.178	.340
20	.056	.091	-1.176	1.020	1.482	.006	1.426	.635
40	.086	.081	-1.705	1.260	1.558	.010	1.980	1.203
60	.114	.078	-2.172	1.389	1.587	.013	2.566	1.763
80	.142	.076	-2.616	1.472	1.602	.016	3.166	2.320
100	.169	.074	-3.047	1.529	1.612	.019	3.772	2.876

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .010 ρ_ℓ / ρ_v = 2000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.016	.170	-.351	.351	.684	.002	1.014	.061
3	.020	.196	-.447	.444	.837	.002	1.029	.099
4	.023	.196	-.516	.510	.932	.003	1.045	.132
5	.026	.189	-.573	.563	1.001	.003	1.061	.162
6	.028	.182	-.622	.608	1.054	.003	1.077	.191
7	.030	.175	-.666	.647	1.097	.003	1.094	.218
8	.032	.168	-.707	.682	1.132	.004	1.111	.245
9	.034	.163	-.745	.714	1.162	.004	1.128	.270
10	.036	.158	-.780	.744	1.188	.004	1.146	.296
20	.050	.128	-1.068	.955	1.334	.006	1.336	.534
40	.074	.108	-1.507	1.185	1.439	.008	1.756	.981
60	.097	.100	-1.886	1.317	1.482	.011	2.200	1.416
80	.118	.096	-2.240	1.404	1.506	.013	2.656	1.847
100	.140	.093	-2.581	1.466	1.521	.016	3.118	2.276

T A B U L A T I O N   O F   S O L U T I O N S

$C_p \Delta T_s / h_{fg}$ = .010

ρ_ℓ / ρ_v = 3000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.016	.243	-.347	.347	.584	.002	1.014	.060
3	.020	.278	-.440	.437	.720	.002	1.028	.096
4	.023	.274	-.507	.501	.809	.003	1.043	.127
5	.025	.263	-.562	.553	.874	.003	1.057	.156
6	.028	.250	-.609	.596	.926	.003	1.072	.183
7	.030	.239	-.651	.634	.968	.003	1.088	.208
8	.031	.229	-.690	.667	1.004	.003	1.103	.233
9	.033	.220	-.725	.698	1.035	.004	1.119	.257
10	.035	.212	-.759	.726	1.062	.004	1.135	.280
20	.048	.166	-1.027	.927	1.221	.005	1.303	.496
40	.070	.135	-1.427	1.150	1.342	.008	1.670	.893
60	.090	.122	-1.768	1.281	1.395	.010	2.055	1.276
80	.109	.115	-2.084	1.369	1.426	.012	2.451	1.654
100	.127	.111	-2.386	1.433	1.445	.014	2.852	2.029

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .010$ $\rho_\ell / \rho_v = 4000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.015	.316	-.345	.345	.516	.002	1.014	.059
3	.020	.360	-.437	.434	.640	.002	1.028	.095
4	.023	.353	-.503	.498	.722	.003	1.042	.125
5	.025	.337	-.556	.548	.784	.003	1.056	.153
6	.027	.320	-.602	.590	.833	.003	1.070	.179
7	.029	.305	-.644	.627	.874	.003	1.085	.204
8	.031	.291	-.681	.660	.909	.003	1.100	.227
9	.033	.278	-.716	.690	.940	.004	1.115	.250
10	.034	.267	-.748	.717	.966	.004	1.130	.273
20	.047	.205	-1.005	.912	1.129	.005	1.287	.477
40	.067	.161	-1.384	1.130	1.262	.007	1.624	.847
60	.086	.144	-1.703	1.259	1.322	.010	1.978	1.201
80	.103	.135	-1.997	1.347	1.357	.011	2.339	1.548
100	.121	.129	-2.277	1.411	1.380	.013	2.705	1.892

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .010

ρ_ℓ / ρ_v = 5000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.015	.390	-.344	.344	.467	.002	1.014	.059
3	.020	.443	-.435	.433	.581	.002	1.027	.094
4	.023	.434	-.500	.495	.657	.003	1.041	.124
5	.025	.413	-.553	.545	.715	.003	1.055	.151
6	.027	.391	-.599	.587	.762	.003	1.069	.177
7	.029	.371	-.639	.623	.802	.003	1.083	.201
8	.031	.353	-.676	.655	.835	.003	1.098	.224
9	.032	.337	-.710	.685	.865	.004	1.112	.246
10	.034	.323	-.742	.712	.891	.004	1.127	.268
20	.046	.244	-.992	.903	1.054	.005	1.278	.465
40	.066	.188	-1.356	1.117	1.193	.007	1.597	.818
60	.083	.166	-1.661	1.244	1.258	.009	1.929	1.153
80	.100	.154	-1.941	1.332	1.296	.011	2.268	1.481
100	.116	.147	-2.207	1.397	1.322	.013	2.612	1.805

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .020 ρ_ℓ / ρ_v = 1000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.020	.203	-.437	.437	.624	.004	1.028	.095
3	.025	.235	-.557	.551	.765	.006	1.056	.153
4	.029	.234	-.644	.630	.855	.006	1.086	.204
5	.033	.226	-.716	.693	.920	.007	1.116	.250
6	.036	.216	-.778	.745	.970	.008	1.146	.294
7	.038	.208	-.835	.791	1.011	.009	1.177	.337
8	.041	.200	-.887	.831	1.045	.009	1.208	.377
9	.043	.194	-.936	.867	1.074	.010	1.240	.417
10	.046	.188	-.982	.900	1.099	.010	1.272	.457
20	.066	.153	-1.363	1.125	1.239	.015	1.608	.826
40	.102	.130	-1.974	1.346	1.339	.023	2.318	1.521
60	.136	.121	-2.523	1.462	1.379	.030	3.049	2.202
80	.169	.116	-3.047	1.534	1.402	.038	3.788	2.877
100	.202	.113	-3.560	1.583	1.416	.045	4.531	3.550

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .020$ $\rho_\ell / \rho_v = 2000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.019	.386	-.431	.431	.463	.004	1.027	.093
3	.025	.440	-.547	.542	.575	.005	1.054	.148
4	.029	.432	-.631	.619	.650	.006	1.081	.196
5	.032	.413	-.700	.679	.706	.007	1.108	.240
6	.035	.393	-.759	.729	.751	.008	1.136	.280
7	.037	.374	-.812	.773	.788	.008	1.164	.319
8	.040	.357	-.861	.811	.820	.009	1.192	.356
9	.042	.343	-.906	.845	.848	.009	1.220	.393
10	.044	.330	-.948	.877	.872	.010	1.249	.428
20	.062	.256	-1.294	1.090	1.021	.014	1.539	.754
40	.093	.205	-1.831	1.305	1.140	.021	2.138	1.350
60	.122	.185	-2.303	1.421	1.194	.027	2.749	1.924
80	.150	.174	-2.749	1.496	1.225	.033	3.364	2.491
100	.177	.168	-3.182	1.548	1.245	.039	3.982	3.053

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .020$ $\rho_\ell / \rho_v = 3000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.019	.570	-.430	.430	.384	.004	1.026	.092
3	.025	.648	-.545	.540	.478	.005	1.053	.147
4	.028	.634	-.628	.616	.542	.006	1.080	.194
5	.032	.604	-.695	.675	.591	.007	1.106	.236
6	.034	.573	-.753	.725	.631	.008	1.133	.276
7	.037	.544	-.805	.767	.664	.008	1.160	.314
8	.039	.518	-.853	.805	.693	.009	1.187	.350
9	.041	.495	-.897	.839	.718	.009	1.214	.385
10	.043	.475	-.938	.869	.741	.010	1.242	.419
20	.061	.360	-1.272	1.078	.883	.014	1.517	.731
40	.090	.282	-1.781	1.290	1.005	.020	2.077	1.292
60	.117	.250	-2.224	1.405	1.063	.026	2.644	1.827
80	.143	.233	-2.641	1.480	1.097	.032	3.212	2.352
100	.169	.223	-3.043	1.533	1.120	.037	3.782	2.872

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .020$ $\qquad$ $\rho_\ell / \rho_v = 4000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.019	.755	-.429	.429	.334	.004	1.026	.092
3	.024	.856	-.544	.539	.418	.005	1.053	.146
4	.028	.838	-.626	.614	.474	.006	1.079	.193
5	.031	.796	-.693	.673	.518	.007	1.106	.235
6	.034	.754	-.751	.723	.553	.008	1.132	.275
7	.037	.714	-.802	.765	.584	.008	1.159	.312
8	.039	.680	-.849	.802	.610	.009	1.185	.348
9	.041	.649	-.893	.836	.633	.009	1.212	.382
10	.043	.622	-.933	.866	.654	.010	1.239	.416
20	.060	.467	-1.261	1.073	.787	.013	1.507	.720
40	.089	.359	-1.757	1.282	.906	.020	2.048	1.264
60	.115	.316	-2.185	1.397	.965	.025	2.592	1.779
80	.140	.293	-2.587	1.472	1.000	.031	3.137	2.283
100	.164	.278	-2.973	1.525	1.024	.036	3.682	2.781

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .020 ρ_ℓ / ρ_v = 5000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.019	.941	-.429	.429	.300	.004	1.026	.091
3	.024	1.066	-.543	.538	.375	.005	1.052	.146
4	.028	1.042	-.625	.613	.426	.006	1.079	.192
5	.031	.989	-.692	.672	.466	.007	1.105	.234
6	.034	.936	-.749	.721	.499	.008	1.131	.274
7	.037	.886	-.801	.764	.526	.008	1.158	.311
8	.039	.842	-.847	.801	.550	.009	1.184	.346
9	.041	.804	-.891	.834	.572	.009	1.211	.380
10	.043	.770	-.931	.864	.591	.010	1.237	.414
20	.060	.574	-1.255	1.069	.715	.013	1.501	.714
40	.088	.438	-1.743	1.277	.830	.020	2.032	1.248
60	.113	.383	-2.163	1.392	.888	.025	2.563	1.751
80	.138	.354	-2.555	1.467	.924	.031	3.093	2.242
100	.161	.335	-2.932	1.520	.948	.036	3.623	2.727

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .040

ρ_ℓ / ρ_v = 1000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.024	.463	-.538	.539	.414	.011	1.052	.143
3	.031	.531	-.686	.673	.513	.014	1.104	.231
4	.036	.525	-.794	.765	.578	.016	1.156	.306
5	.040	.505	-.883	.836	.627	.018	1.209	.374
6	.044	.482	-.962	.894	.666	.020	1.261	.439
7	.048	.462	-1.033	.943	.698	.021	1.314	.501
8	.051	.443	-1.098	.986	.726	.023	1.367	.561
9	.054	.427	-1.160	1.024	.750	.024	1.420	.620
10	.058	.412	-1.218	1.058	.770	.026	1.473	.677
20	.085	.330	-1.712	1.276	.895	.038	2.006	1.211
40	.135	.274	-2.528	1.472	.992	.060	3.077	2.208
60	.182	.253	-3.278	1.568	1.034	.081	4.151	3.178
80	.229	.241	-4.003	1.626	1.058	.102	5.225	4.139
100	.275	.234	-4.716	1.665	1.073	.122	6.299	5.094

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .040$ $\rho_\ell / \rho_v = 2000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.024	.912	-.536	.537	.297	.011	1.051	.142
3	.031	1.043	-.683	.670	.370	.014	1.103	.228
4	.036	1.027	-.789	.761	.418	.016	1.154	.302
5	.040	.983	-.877	.831	.456	.018	1.205	.370
6	.044	.937	-.954	.888	.486	.019	1.256	.433
7	.047	.893	-1.023	.937	.511	.021	1.307	.493
8	.051	.855	-1.087	.979	.533	.023	1.358	.551
9	.054	.821	-1.147	1.016	.552	.024	1.409	.608
10	.057	.791	-1.204	1.050	.569	.025	1.460	.663
20	.083	.619	-1.679	1.265	.674	.037	1.966	1.173
40	.130	.501	-2.451	1.459	.762	.058	2.971	2.111
60	.174	.455	-3.154	1.555	.804	.078	3.970	3.015
80	.218	.429	-3.829	1.614	.828	.097	4.966	3.907
100	.261	.414	-4.492	1.654	.844	.116	5.960	4.793

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .040$ $\qquad$ $\rho_\ell / \rho_v = 3000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.024	1.363	-.536	.536	.243	.011	1.051	.142
3	.031	1.556	-.682	.670	.303	.014	1.102	.228
4	.036	1.531	-.788	.760	.344	.016	1.153	.301
5	.040	1.465	-.876	.830	.375	.018	1.204	.368
6	.044	1.394	-.952	.887	.400	.019	1.255	.431
7	.047	1.328	-1.021	.935	.421	.021	1.305	.491
8	.051	1.270	-1.084	.977	.439	.022	1.356	.549
9	.054	1.218	-1.144	1.014	.455	.024	1.406	.605
10	.057	1.173	-1.200	1.048	.470	.025	1.456	.659
20	.083	.912	-1.669	1.261	.560	.037	1.955	1.162
40	.129	.732	-2.429	1.455	.639	.057	2.940	2.083
60	.172	.661	-3.117	1.551	.676	.077	3.917	2.968
80	.215	.622	-3.778	1.610	.699	.095	4.889	3.839
100	.257	.597	-4.425	1.650	.714	.114	5.859	4.703

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .040 ρ_ℓ / ρ_v = 4000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.024	1.814	-.536	.536	.211	.011	1.051	.142
3	.031	2.071	-.681	.669	.263	.014	1.102	.228
4	.036	2.037	-.787	.760	.299	.016	1.153	.301
5	.040	1.947	-.875	.829	.326	.018	1.203	.368
6	.044	1.852	-.951	.886	.348	.019	1.254	.430
7	.047	1.764	-1.020	.935	.366	.021	1.304	.490
8	.050	1.686	-1.083	.977	.382	.022	1.355	.548
9	.054	1.617	-1.142	1.014	.396	.024	1.405	.603
10	.057	1.556	-1.199	1.047	.409	.025	1.455	.658
20	.083	1.206	-1.665	1.260	.489	.037	1.950	1.158
40	.128	.965	-2.419	1.453	.560	.057	2.927	2.071
60	.171	.868	-3.101	1.549	.594	.076	3.894	2.947
80	.213	.816	-3.755	1.608	.615	.095	4.855	3.808
100	.255	.783	-4.395	1.649	.629	.113	5.814	4.663

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .040

ρ_ℓ / ρ_v = 5000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.024	2.266	-.536	.536	.189	.011	1.051	.142
3	.031	2.585	-.681	.669	.236	.014	1.102	.227
4	.036	2.542	-.787	.760	.267	.016	1.153	.301
5	.040	2.430	-.874	.829	.292	.018	1.203	.367
6	.044	2.311	-.951	.886	.311	.019	1.254	.430
7	.047	2.201	-1.019	.934	.328	.021	1.304	.490
8	.050	2.103	-1.083	.976	.343	.022	1.354	.547
9	.054	2.017	-1.142	1.013	.355	.024	1.404	.603
10	.056	1.941	-1.198	1.046	.367	.025	1.454	.657
20	.083	1.501	-1.663	1.259	.439	.037	1.948	1.155
40	.128	1.198	-2.414	1.452	.504	.057	2.920	2.064
60	.171	1.077	-3.092	1.549	.535	.076	3.881	2.935
80	.212	1.011	-3.743	1.608	.555	.094	4.837	3.792
100	.253	.969	-4.379	1.648	.568	.113	5.789	4.641

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .060 ρ_ℓ / ρ_v = 1000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.027	.755	-.610	.611	.318	.018	1.075	.183
3	.035	.871	-.780	.760	.394	.023	1.151	.295
4	.041	.865	-.905	.860	.444	.027	1.226	.392
5	.046	.834	-1.010	.936	.481	.031	1.301	.481
6	.051	.800	-1.103	.997	.511	.034	1.375	.566
7	.055	.768	-1.187	1.049	.536	.037	1.450	.646
8	.060	.739	-1.266	1.093	.557	.040	1.525	.725
9	.064	.713	-1.341	1.131	.576	.042	1.599	.801
10	.068	.691	-1.412	1.165	.592	.045	1.673	.876
20	.103	.562	-2.024	1.378	.688	.069	2.412	1.580
40	.168	.474	-3.069	1.556	.763	.112	3.875	2.904
60	.231	.440	-4.047	1.638	.797	.154	5.330	4.196
80	.292	.422	-5.000	1.686	.816	.195	6.782	5.476
100	.354	.410	-5.942	1.718	.828	.236	8.231	6.750

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .060$ $\rho_\ell / \rho_v = 2000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.027	1.500	-.609	.610	.226	.018	1.075	.183
3	.035	1.726	-.778	.759	.281	.023	1.150	.294
4	.041	1.711	-.903	.858	.317	.027	1.224	.390
5	.046	1.647	-1.007	.934	.345	.031	1.298	.478
6	.051	1.577	-1.099	.995	.367	.034	1.372	.562
7	.055	1.512	-1.182	1.046	.385	.037	1.445	.642
8	.059	1.453	-1.260	1.090	.401	.040	1.519	.719
9	.063	1.401	-1.334	1.128	.415	.042	1.592	.794
10	.067	1.355	-1.404	1.162	.427	.045	1.665	.868
20	.102	1.091	-2.005	1.373	.501	.068	2.387	1.557
40	.165	.911	-3.024	1.550	.562	.110	3.809	2.845
60	.226	.839	-3.973	1.633	.590	.151	5.219	4.097
80	.285	.801	-4.896	1.682	.606	.190	6.623	5.336
100	.345	.776	-5.807	1.714	.617	.230	8.023	6.568

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .060$ $\rho_\ell / \rho_v = 3000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.027	2.247	-.609	.609	.185	.018	1.075	.182
3	.035	2.584	-.778	.758	.230	.023	1.149	.293
4	.041	2.560	-.902	.858	.260	.027	1.224	.389
5	.046	2.463	-1.006	.933	.283	.031	1.297	.478
6	.051	2.358	-1.098	.994	.301	.034	1.371	.561
7	.055	2.259	-1.181	1.045	.316	.037	1.444	.640
8	.059	2.170	-1.259	1.089	.329	.040	1.517	.717
9	.063	2.092	-1.332	1.127	.340	.042	1.590	.792
10	.067	2.023	-1.402	1.161	.351	.045	1.663	.866
20	.102	1.624	-2.000	1.372	.413	.068	2.380	1.551
40	.164	1.351	-3.012	1.549	.464	.110	3.791	2.829
60	.224	1.243	-3.953	1.632	.488	.150	5.189	4.070
80	.284	1.184	-4.868	1.681	.502	.189	6.580	5.298
100	.342	1.147	-5.771	1.713	.511	.228	7.967	6.518

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .060$ $\rho_\ell / \rho_v = 4000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.027	2.993	-.609	.609	.160	.018	1.075	.182
3	.035	3.442	-.777	.758	.199	.023	1.149	.293
4	.041	3.409	-.902	.857	.225	.027	1.223	.389
5	.046	3.280	-1.006	.933	.245	.031	1.297	.477
6	.051	3.139	-1.097	.994	.261	.034	1.371	.560
7	.055	3.007	-1.181	1.045	.274	.037	1.444	.640
8	.059	2.889	-1.258	1.088	.286	.040	1.517	.717
9	.063	2.784	-1.331	1.127	.296	.042	1.589	.791
10	.067	2.692	-1.401	1.160	.304	.045	1.662	.865
20	.102	2.159	-1.998	1.371	.359	.068	2.377	1.549
40	.164	1.793	-3.007	1.548	.404	.109	3.784	2.823
60	.224	1.648	-3.944	1.631	.425	.149	5.176	4.059
80	.283	1.569	-4.856	1.680	.437	.189	6.561	5.281
100	.341	1.519	-5.755	1.713	.445	.228	7.943	6.497

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .060$

$\rho_\ell / \rho_v = 5000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.027	3.740	-.608	.609	.143	.018	1.075	.182
3	.035	4.301	-.777	.758	.178	.023	1.149	.293
4	.041	4.259	-.902	.857	.202	.027	1.223	.389
5	.046	4.098	-1.005	.933	.219	.031	1.297	.477
6	.051	3.921	-1.097	.993	.234	.034	1.370	.560
7	.055	3.756	-1.180	1.044	.245	.037	1.443	.639
8	.059	3.607	-1.258	1.088	.256	.039	1.516	.716
9	.063	3.477	-1.331	1.126	.265	.042	1.589	.791
10	.067	3.361	-1.400	1.160	.273	.045	1.661	.864
20	.102	2.694	-1.997	1.371	.321	.068	2.376	1.547
40	.164	2.235	-3.004	1.548	.362	.109	3.780	2.819
60	.224	2.053	-3.940	1.631	.381	.149	5.169	4.053
80	.282	1.955	-4.850	1.680	.392	.188	6.551	5.273
100	.341	1.893	-5.747	1.713	.400	.227	7.930	6.485

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .080

ρ_ℓ / ρ_v = 1000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.029	1.063	-.666	.667	.262	.026	1.098	.217
3	.038	1.232	-.854	.827	.324	.034	1.196	.351
4	.045	1.230	-.995	.933	.364	.040	1.293	.468
5	.051	1.192	-1.114	1.012	.395	.045	1.389	.576
6	.057	1.149	-1.220	1.075	.419	.050	1.486	.678
7	.062	1.107	-1.317	1.128	.438	.055	1.581	.776
8	.067	1.069	-1.408	1.173	.455	.059	1.677	.872
9	.072	1.036	-1.495	1.211	.470	.064	1.772	.966
10	.076	1.007	-1.578	1.245	.482	.068	1.868	1.058
20	.120	.836	-2.307	1.451	.557	.106	2.809	1.928
40	.200	.720	-3.584	1.614	.614	.177	4.669	3.579
60	.277	.675	-4.794	1.687	.639	.246	6.515	5.196
80	.354	.651	-5.982	1.729	.654	.315	8.356	6.802
100	.431	.635	-7.159	1.756	.663	.383	10.194	8.402

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .080$ $\qquad\qquad$ $\rho_\ell / \rho_v = 2000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.029	2.117	-.665	.667	.186	.026	1.098	.217
3	.038	2.453	-.853	.826	.230	.034	1.195	.350
4	.045	2.446	-.993	.932	.259	.040	1.292	.466
5	.051	2.368	-1.112	1.011	.281	.045	1.388	.574
6	.057	2.279	-1.217	1.074	.298	.050	1.483	.675
7	.062	2.195	-1.314	1.126	.313	.055	1.578	.773
8	.067	2.119	-1.404	1.171	.325	.059	1.673	.868
9	.072	2.051	-1.490	1.209	.336	.064	1.767	.961
10	.076	1.992	-1.573	1.243	.345	.068	1.862	1.052
20	.119	1.646	-2.295	1.448	.401	.106	2.792	1.913
40	.198	1.410	-3.554	1.612	.444	.176	4.624	3.540
60	.274	1.317	-4.746	1.685	.464	.244	6.441	5.132
80	.350	1.267	-5.915	1.727	.475	.311	8.251	6.711
100	.425	1.235	-7.072	1.754	.482	.378	10.058	8.284

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .080 ρ_ℓ / ρ_v = 3000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.029	3.172	-.665	.667	.152	.026	1.098	.217
3	.038	3.675	-.853	.826	.188	.034	1.195	.350
4	.045	3.664	-.993	.931	.212	.040	1.291	.466
5	.051	3.546	-1.111	1.010	.230	.045	1.387	.573
6	.057	3.412	-1.216	1.073	.244	.050	1.482	.675
7	.062	3.285	-1.313	1.126	.256	.055	1.577	.772
8	.067	3.171	-1.403	1.170	.266	.059	1.672	.867
9	.071	3.069	-1.489	1.209	.275	.064	1.766	.960
10	.076	2.979	-1.571	1.243	.282	.068	1.860	1.050
20	.119	2.459	-2.292	1.448	.328	.105	2.788	1.909
40	.197	2.103	-3.547	1.611	.365	.175	4.613	3.530
60	.273	1.962	-4.734	1.684	.381	.243	6.422	5.115
80	.349	1.886	-5.897	1.726	.390	.310	8.224	6.687
100	.424	1.839	-7.049	1.754	.396	.376	10.023	8.253

T A B U L A T I O N   O F   S O L U T I O N S

$C_p \Delta T_s / h_{fg} = .080$ $\qquad\qquad$ $\rho_\ell / \rho_v = 4000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.029	4.228	-.665	.666	.132	.026	1.098	.217
3	.038	4.897	-.853	.826	.163	.034	1.195	.350
4	.045	4.882	-.993	.931	.184	.040	1.291	.466
5	.051	4.725	-1.111	1.010	.199	.045	1.387	.573
6	.057	4.546	-1.216	1.073	.212	.050	1.482	.674
7	.062	4.377	-1.313	1.126	.222	.055	1.577	.772
8	.067	4.224	-1.403	1.170	.231	.059	1.671	.867
9	.071	4.088	-1.489	1.209	.238	.063	1.766	.959
10	.076	3.968	-1.571	1.243	.245	.068	1.859	1.050
20	.119	3.273	-2.290	1.447	.285	.105	2.786	1.907
40	.197	2.797	-3.544	1.611	.317	.175	4.608	3.526
60	.273	2.609	-4.729	1.684	.331	.243	6.414	5.108
80	.348	2.508	-5.890	1.726	.339	.309	8.213	6.677
100	.423	2.444	-7.039	1.754	.344	.376	10.008	8.240

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .080

ρ_ℓ / ρ_v = 5000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.029	5.283	-.665	.666	.118	.026	1.098	.217
3	.038	6.119	-.853	.826	.146	.034	1.195	.350
4	.045	6.101	-.993	.931	.164	.040	1.291	.466
5	.051	5.904	-1.111	1.010	.178	.045	1.387	.573
6	.057	5.680	-1.216	1.073	.189	.050	1.482	.674
7	.062	5.468	-1.312	1.126	.199	.055	1.577	.772
8	.067	5.277	-1.403	1.170	.206	.059	1.671	.866
9	.071	5.107	-1.488	1.209	.213	.063	1.765	.959
10	.076	4.957	-1.571	1.242	.219	.068	1.859	1.050
20	.119	4.087	-2.289	1.447	.255	.105	2.785	1.906
40	.197	3.491	-3.542	1.611	.284	.175	4.606	3.523
60	.273	3.256	-4.726	1.684	.296	.242	6.410	5.104
80	.348	3.130	-5.886	1.726	.304	.309	8.206	6.671
100	.423	3.050	-7.034	1.754	.309	.376	10.000	8.233

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .100$ $\rho_\ell / \rho_v = 1000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.031	1.375	-.712	.714	.225	.035	1.120	.247
3	.041	1.605	-.916	.883	.277	.045	1.239	.401
4	.049	1.612	-1.071	.992	.311	.054	1.357	.535
5	.055	1.569	-1.202	1.074	.337	.062	1.475	.660
6	.062	1.518	-1.320	1.138	.356	.068	1.591	.779
7	.068	1.469	-1.430	1.191	.373	.075	1.708	.894
8	.073	1.424	-1.532	1.236	.386	.081	1.824	1.006
9	.079	1.384	-1.630	1.274	.398	.088	1.939	1.116
10	.084	1.349	-1.725	1.308	.408	.094	2.055	1.224
20	.135	1.143	-2.567	1.506	.468	.150	3.195	2.254
40	.229	1.003	-4.070	1.658	.513	.255	5.444	4.223
60	.322	.948	-5.509	1.723	.532	.357	7.677	6.158
80	.413	.919	-6.927	1.760	.542	.459	9.905	8.083
100	.504	.900	-8.334	1.784	.549	.561	12.130	10.002

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .100 $\qquad\qquad$ ρ_ℓ / ρ_v = 2000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.031	2.744	-.711	.714	.159	.035	1.120	.247
3	.041	3.200	-.916	.882	.197	.045	1.238	.400
4	.048	3.211	-1.070	.991	.221	.054	1.356	.534
5	.055	3.125	-1.201	1.073	.239	.061	1.473	.659
6	.062	3.022	-1.319	1.137	.253	.068	1.590	.777
7	.067	2.922	-1.427	1.190	.265	.075	1.705	.892
8	.073	2.832	-1.530	1.235	.275	.081	1.821	1.003
9	.079	2.751	-1.627	1.273	.283	.087	1.936	1.113
10	.084	2.680	-1.721	1.307	.291	.093	2.050	1.220
20	.134	2.264	-2.558	1.505	.334	.149	3.182	2.243
40	.228	1.980	-4.049	1.657	.367	.253	5.412	4.195
60	.319	1.869	-5.476	1.722	.382	.355	7.625	6.113
80	.410	1.809	-6.880	1.759	.390	.456	9.831	8.019
100	.501	1.771	-8.274	1.783	.395	.556	12.035	9.920

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .100 $\qquad\qquad$ ρ_ℓ / ρ_v = 3000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.031	4.113	-.711	.714	.130	.035	1.120	.247
3	.041	4.797	-.915	.882	.161	.045	1.238	.400
4	.048	4.812	-1.070	.991	.181	.054	1.356	.534
5	.055	4.682	-1.201	1.072	.195	.061	1.473	.658
6	.062	4.527	-1.318	1.137	.207	.068	1.589	.777
7	.067	4.377	-1.427	1.190	.217	.075	1.705	.891
8	.073	4.241	-1.529	1.234	.225	.081	1.820	1.003
9	.079	4.120	-1.627	1.273	.232	.087	1.935	1.112
10	.084	4.013	-1.720	1.306	.238	.093	2.049	1.219
20	.134	3.388	-2.556	1.505	.274	.149	3.179	2.240
40	.228	2.960	-4.044	1.656	.301	.253	5.404	4.188
60	.319	2.792	-5.467	1.722	.313	.354	7.612	6.102
80	.409	2.702	-6.868	1.759	.320	.455	9.813	8.003
100	.500	2.646	-8.259	1.783	.324	.555	12.010	9.899

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .100$

$\rho_\ell / \rho_v = 4000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.031	5.482	-.711	.714	.113	.035	1.120	.247
3	.041	6.394	-.915	.882	.139	.045	1.238	.400
4	.048	6.413	-1.069	.991	.157	.054	1.356	.534
5	.055	6.240	-1.200	1.072	.169	.061	1.473	.658
6	.062	6.033	-1.318	1.137	.179	.068	1.589	.777
7	.067	5.833	-1.426	1.190	.188	.075	1.704	.891
8	.073	5.652	-1.529	1.234	.195	.081	1.820	1.002
9	.079	5.490	-1.626	1.273	.201	.087	1.934	1.111
10	.084	5.346	-1.720	1.306	.206	.093	2.049	1.219
20	.134	4.513	-2.555	1.505	.237	.149	3.178	2.239
40	.227	3.941	-4.042	1.656	.261	.253	5.400	4.185
60	.319	3.717	-5.464	1.722	.271	.354	7.606	6.097
80	.409	3.596	-6.863	1.759	.277	.455	9.805	7.996
100	.499	3.521	-8.252	1.783	.281	.555	12.000	9.890

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .100$ $\rho_\ell / \rho_v = 5000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.000	0.000	0.000	0.000	0.000	1.000	0.000
2	.031	6.852	-.711	.714	.101	.035	1.120	.247
3	.041	7.991	-.915	.882	.125	.045	1.238	.400
4	.048	8.015	-1.069	.991	.140	.054	1.356	.534
5	.055	7.798	-1.200	1.072	.152	.061	1.473	.658
6	.061	7.539	-1.318	1.137	.161	.068	1.589	.777
7	.067	7.289	-1.426	1.189	.168	.075	1.704	.891
8	.073	7.062	-1.529	1.234	.174	.081	1.819	1.002
9	.079	6.860	-1.626	1.273	.180	.087	1.934	1.111
10	.084	6.680	-1.720	1.306	.184	.093	2.048	1.218
20	.134	5.638	-2.555	1.504	.212	.149	3.177	2.239
40	.227	4.923	-4.041	1.656	.234	.253	5.399	4.184
60	.319	4.642	-5.462	1.722	.243	.354	7.603	6.094
80	.409	4.492	-6.861	1.759	.248	.454	9.800	7.992
100	.499	4.397	-8.249	1.783	.252	.554	11.994	9.885

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .120$ $\rho_\ell / \rho_v = 1000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.033	1.69	-.751	.754	.198	.044	1.141	.274
3	.043	1.98	-.970	.929	.244	.057	1.280	.445
4	.052	2.00	-1.137	1.042	.273	.069	1.419	.596
5	.059	1.96	-1.280	1.125	.295	.079	1.556	.737
6	.066	1.90	-1.409	1.190	.311	.088	1.693	.872
7	.073	1.85	-1.530	1.243	.325	.097	1.829	1.003
8	.079	1.80	-1.643	1.288	.337	.105	1.965	1.130
9	.085	1.75	-1.752	1.326	.346	.114	2.100	1.255
10	.091	1.71	-1.858	1.359	.355	.122	2.235	1.379
20	.149	1.48	-2.808	1.551	.404	.198	3.566	2.561
40	.257	1.31	-4.529	1.693	.440	.343	6.195	4.834
60	.363	1.25	-6.189	1.752	.454	.485	8.807	7.076
80	.469	1.22	-7.829	1.786	.463	.625	11.413	9.307
100	.574	1.20	-9.459	1.807	.468	.766	14.016	11.933

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .120 ρ_ℓ / ρ_v = 2000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.033	3.37	-.751	.754	.140	.044	1.141	.274
3	.043	3.96	-.969	.929	.173	.057	1.280	.445
4	.051	3.99	-1.136	1.042	.194	.069	1.418	.596
5	.059	3.90	-1.279	1.124	.209	.079	1.555	.736
6	.066	3.79	-1.408	1.189	.221	.088	1.692	.871
7	.072	3.68	-1.528	1.242	.231	.097	1.827	1.001
8	.079	3.58	-1.641	1.287	.239	.105	1.962	1.128
9	.085	3.49	-1.750	1.325	.246	.113	2.097	1.253
10	.091	3.41	-1.855	1.358	.252	.122	2.231	1.375
20	.148	2.93	-2.801	1.550	.288	.198	3.557	2.552
40	.256	2.61	-4.514	1.692	.314	.341	6.171	4.814
60	.362	2.48	-6.164	1.752	.325	.482	8.768	7.042
80	.467	2.41	-7.795	1.785	.331	.622	11.358	9.260
100	.572	2.37	-9.415	1.807	.335	.762	13.946	11.473

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .120$ $\rho_\ell / \rho_v = 3000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.033	5.05	-.751	.754	.115	.044	1.141	.274
3	.043	5.93	-.969	.929	.141	.057	1.280	.445
4	.051	5.98	-1.136	1.041	.158	.069	1.418	.595
5	.059	5.85	-1.279	1.124	.171	.079	1.555	.736
6	.066	5.68	-1.408	1.189	.181	.088	1.691	.870
7	.072	5.51	-1.528	1.242	.189	.097	1.827	1.000
8	.079	5.36	-1.641	1.287	.195	.105	1.962	1.127
9	.085	5.22	-1.749	1.325	.201	.113	2.096	1.252
10	.091	5.10	-1.854	1.358	.206	.121	2.230	1.375
20	.148	4.39	-2.799	1.550	.235	.197	3.554	2.550
40	.256	3.90	-4.510	1.692	.257	.341	6.165	4.808
60	.361	3.71	-6.158	1.752	.266	.482	8.758	7.034
80	.466	3.61	-7.786	1.785	.271	.622	11.344	9.248
100	.571	3.54	-9.404	1.806	.274	.761	13.928	11.458

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .120

ρ_ℓ / ρ_v = 4000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.033	6.74	-.751	.754	.099	.044	1.141	.274
3	.043	7.91	-.969	.929	.122	.057	1.280	.445
4	.051	7.97	-1.136	1.041	.137	.069	1.418	.595
5	.059	7.80	-1.279	1.124	.148	.079	1.555	.736
6	.066	7.57	-1.408	1.189	.157	.088	1.691	.870
7	.072	7.35	-1.527	1.242	.164	.097	1.827	1.000
8	.079	7.14	-1.641	1.287	.169	.105	1.961	1.127
9	.085	6.96	-1.749	1.325	.174	.113	2.096	1.252
10	.091	6.80	-1.854	1.358	.179	.121	2.230	1.374
20	.148	5.85	-2.799	1.550	.204	.197	3.553	2.549
40	.256	5.20	-4.508	1.691	.222	.341	6.162	4.806
60	.361	4.94	-6.156	1.751	.230	.482	8.754	7.030
80	.466	4.81	-7.782	1.785	.235	.621	11.339	9.243
100	.571	4.72	-9.400	1.806	.237	.761	13.920	11.451

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .120

ρ_ℓ / ρ_v = 5000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.033	8.42	-.751	.754	.089	.044	1.141	.274
3	.043	9.88	-.969	.929	.109	.057	1.280	.445
4	.051	9.97	-1.136	1.041	.123	.069	1.418	.595
5	.059	9.75	-1.279	1.124	.132	.079	1.555	.736
6	.066	9.46	-1.407	1.189	.140	.088	1.691	.870
7	.072	9.18	-1.527	1.242	.146	.097	1.826	1.000
8	.079	8.93	-1.641	1.287	.152	.105	1.961	1.127
9	.085	8.70	-1.749	1.325	.156	.113	2.096	1.251
10	.091	8.50	-1.854	1.358	.160	.121	2.230	1.374
20	.148	7.31	-2.798	1.550	.182	.197	3.553	2.549
40	.256	6.49	-4.507	1.691	.199	.341	6.161	4.805
60	.361	6.17	-6.154	1.751	.206	.482	8.752	7.029
80	.466	6.00	-7.780	1.785	.210	.621	11.336	9.241
100	.570	5.90	-9.397	1.806	.213	.761	13.916	11.448

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .140 ρ_ℓ / ρ_v = 1000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.034	2.00	-.785	.790	.178	.053	1.161	.298
3	.045	2.36	-1.017	.970	.218	.070	1.320	.486
4	.054	2.39	-1.195	1.085	.244	.084	1.478	.652
5	.062	2.35	-1.350	1.169	.263	.097	1.635	.808
6	.070	2.29	-1.489	1.234	.277	.109	1.791	.958
7	.077	2.23	-1.620	1.287	.289	.120	1.946	1.103
8	.084	2.18	-1.744	1.332	.299	.131	2.100	1.245
9	.091	2.13	-1.864	1.369	.307	.142	2.254	1.384
10	.098	2.09	-1.979	1.402	.315	.152	2.407	1.522
20	.162	1.83	-3.032	1.588	.356	.251	3.924	2.849
40	.283	1.65	-4.963	1.721	.385	.440	6.921	5.414
60	.403	1.58	-6.835	1.776	.397	.626	9.900	7.948
80	.522	1.54	-8.687	1.807	.403	.811	12.874	10.472
100	.640	1.52	-10.532	1.826	.408	.996	15.845	12.992

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .140

ρ_ℓ / ρ_v = 2000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.034	3.99	-.785	.790	.126	.053	1.161	.298
3	.045	4.71	-1.017	.970	.155	.070	1.320	.485
4	.054	4.78	-1.195	1.085	.173	.084	1.478	.652
5	.062	4.69	-1.349	1.169	.186	.097	1.634	.807
6	.070	4.57	-1.488	1.234	.197	.109	1.790	.956
7	.077	4.46	-1.619	1.287	.205	.120	1.944	1.101
8	.084	4.34	-1.743	1.331	.212	.131	2.098	1.243
9	.091	4.25	-1.862	1.369	.218	.141	2.252	1.382
10	.098	4.16	-1.977	1.401	.223	.152	2.405	1.520
20	.161	3.64	-3.027	1.587	.253	.251	3.917	2.842
40	.282	3.28	-4.951	1.721	.274	.439	6.902	5.398
60	.401	3.14	-6.816	1.776	.283	.625	9.870	7.922
80	.520	3.06	-8.661	1.806	.287	.809	12.832	10.436
100	.638	3.02	-10.498	1.826	.290	.993	15.791	12.946

T A B U L A T I O N   O F   S O L U T I O N S

$C_p \Delta T_s / h_{fg} = .140$ $\rho_\ell / \rho_v = 3000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.034	5.98	-.785	.789	.103	.053	1.161	.298
3	.045	7.06	-1.016	.970	.126	.070	1.320	.485
4	.054	7.16	-1.195	1.085	.141	.084	1.477	.651
5	.062	7.03	-1.348	1.168	.152	.097	1.634	.807
6	.070	6.86	-1.488	1.234	.161	.109	1.789	.956
7	.077	6.68	-1.618	1.287	.168	.120	1.944	1.101
8	.084	6.51	-1.742	1.331	.173	.131	2.098	1.242
9	.091	6.36	-1.861	1.369	.178	.141	2.251	1.382
10	.098	6.23	-1.976	1.401	.183	.152	2.404	1.519
20	.161	5.45	-3.026	1.587	.207	.251	3.915	2.841
40	.282	4.91	-4.948	1.720	.224	.439	6.897	5.394
60	.401	4.70	-6.811	1.776	.231	.624	9.863	7.916
80	.520	4.59	-8.655	1.806	.235	.808	12.821	10.427
100	.638	4.52	-10.490	1.826	.238	.992	15.777	12.934

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .140 ρ_ℓ / ρ_v = 4000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.034	7.98	-.785	.789	.089	.053	1.161	.298
3	.045	9.42	-1.016	.970	.109	.070	1.320	.485
4	.054	9.55	-1.195	1.085	.122	.084	1.477	.651
5	.062	9.38	-1.348	1.168	.132	.097	1.634	.807
6	.070	9.14	-1.488	1.234	.139	.108	1.789	.956
7	.077	8.90	-1.618	1.287	.145	.120	1.944	1.101
8	.084	8.68	-1.742	1.331	.150	.131	2.098	1.242
9	.091	8.48	-1.861	1.369	.154	.141	2.251	1.381
10	.098	8.30	-1.976	1.401	.158	.152	2.404	1.519
20	.161	7.26	-3.025	1.587	.179	.251	3.914	2.840
40	.282	6.54	-4.947	1.720	.194	.439	6.895	5.392
60	.401	6.26	-6.809	1.776	.200	.624	9.859	7.913
80	.519	6.11	-8.652	1.806	.204	.808	12.817	10.423
100	.637	6.02	-10.486	1.826	.206	.991	15.771	12.929

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .140$ $\rho_\ell / \rho_v = 5000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.034	9.97	-.785	.789	.080	.053	1.161	.298
3	.045	11.77	-1.016	.970	.098	.070	1.320	.485
4	.054	11.93	-1.195	1.085	.110	.084	1.477	.651
5	.062	11.72	-1.348	1.168	.118	.097	1.634	.807
6	.070	11.42	-1.488	1.234	.125	.108	1.789	.956
7	.077	11.13	-1.618	1.287	.130	.120	1.944	1.101
8	.084	10.85	-1.742	1.331	.134	.131	2.097	1.242
9	.091	10.60	-1.861	1.369	.138	.141	2.251	1.381
10	.098	10.38	-1.976	1.401	.141	.152	2.403	1.519
20	.161	9.07	-3.025	1.587	.160	.251	3.914	2.840
40	.282	8.17	-4.946	1.720	.174	.439	6.894	5.391
60	.401	7.82	-6.808	1.776	.179	.624	9.858	7.911
80	.519	7.64	-8.651	1.806	.182	.808	12.815	10.421
100	.637	7.52	-10.484	1.826	.184	.991	15.768	12.927

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .160$ $\rho_\ell / \rho_v = 1000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.035	2.30	-.815	.821	.162	.062	1.180	.320
3	.047	2.73	-1.059	1.006	.198	.083	1.359	.523
4	.056	2.79	-1.248	1.123	.221	.100	1.535	.704
5	.065	2.75	-1.412	1.208	.238	.116	1.710	.874
6	.073	2.69	-1.562	1.273	.251	.130	1.885	1.037
7	.081	2.63	-1.703	1.326	.261	.144	2.058	1.196
8	.089	2.57	-1.837	1.370	.270	.158	2.230	1.351
9	.096	2.52	-1.966	1.407	.277	.171	2.402	1.505
10	.104	2.47	-2.091	1.439	.283	.184	2.574	1.656
20	.173	2.19	-3.242	1.619	.318	.308	4.270	3.121
40	.307	2.00	-5.373	1.745	.342	.547	7.622	5.962
60	.440	1.93	-7.447	1.797	.352	.781	10.958	8.775
80	.571	1.89	-9.504	1.825	.358	1.015	14.288	11.579
100	.702	1.86	-11.552	1.843	.361	1.249	17.615	14.378

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .160 $\qquad\qquad$ ρ_ℓ / ρ_v = 2000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.035	4.60	-.815	.821	.115	.062	1.180	.320
3	.047	5.46	-1.059	1.006	.140	.083	1.358	.522
4	.056	5.56	-1.248	1.123	.157	.100	1.535	.703
5	.065	5.49	-1.412	1.207	.169	.116	1.710	.873
6	.073	5.37	-1.561	1.273	.178	.130	1.884	1.036
7	.081	5.24	-1.702	1.325	.185	.144	2.057	1.195
8	.089	5.13	-1.835	1.369	.191	.158	2.229	1.350
9	.096	5.02	-1.964	1.406	.196	.171	2.400	1.503
10	.104	4.93	-2.089	1.438	.201	.184	2.571	1.654
20	.173	4.37	-3.238	1.618	.226	.308	4.263	3.115
40	.307	3.98	-5.364	1.745	.243	.546	7.607	5.950
60	.439	3.83	-7.433	1.796	.250	.780	10.934	8.755
80	.570	3.75	-9.483	1.825	.254	1.013	14.254	11.550
100	.701	3.70	-11.526	1.842	.257	1.245	17.572	14.342

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .160$ $\rho_\ell / \rho_v = 3000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.035	6.89	-.815	.821	.094	.062	1.180	.320
3	.047	8.19	-1.058	1.006	.115	.083	1.358	.522
4	.056	8.34	-1.247	1.123	.128	.100	1.535	.703
5	.065	8.22	-1.411	1.207	.138	.116	1.709	.873
6	.073	8.04	-1.561	1.273	.145	.130	1.883	1.036
7	.081	7.86	-1.701	1.325	.151	.144	2.056	1.194
8	.089	7.68	-1.835	1.369	.156	.158	2.228	1.350
9	.096	7.53	-1.964	1.406	.160	.171	2.400	1.503
10	.103	7.38	-2.089	1.438	.164	.184	2.571	1.654
20	.173	6.55	-3.237	1.618	.184	.308	4.262	3.114
40	.307	5.97	-5.361	1.745	.199	.545	7.603	5.947
60	.438	5.74	-7.429	1.796	.205	.779	10.928	8.750
80	.569	5.62	-9.478	1.825	.208	1.012	14.246	11.543
100	.700	5.55	-11.519	1.842	.210	1.245	17.561	14.333

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .160 ρ_ℓ / ρ_v = 4000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.035	9.19	-.815	.821	.081	.062	1.180	.320
3	.047	10.91	-1.058	1.006	.099	.083	1.358	.522
4	.056	11.12	-1.247	1.123	.111	.100	1.535	.703
5	.065	10.96	-1.411	1.207	.119	.116	1.709	.873
6	.073	10.72	-1.561	1.273	.126	.130	1.883	1.036
7	.081	10.48	-1.701	1.325	.131	.144	2.056	1.194
8	.089	10.24	-1.835	1.369	.135	.158	2.228	1.349
9	.096	10.03	-1.964	1.406	.139	.171	2.400	1.502
10	.103	9.84	-2.089	1.438	.142	.184	2.571	1.654
20	.173	8.72	-3.237	1.618	.160	.308	4.261	3.114
40	.307	7.95	-5.360	1.745	.172	.545	7.602	5.945
60	.438	7.65	-7.427	1.796	.177	.779	10.925	8.748
80	.569	7.49	-9.476	1.824	.180	1.012	14.242	11.540
100	.700	7.39	-11.517	1.842	.182	1.244	17.557	14.329

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .160$ $\rho_\ell / \rho_v = 5000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.035	11.49	-.815	.821	.073	.062	1.180	.320
3	.047	13.64	-1.058	1.006	.089	.083	1.358	.522
4	.056	13.90	-1.247	1.123	.099	.100	1.534	.703
5	.065	13.70	-1.411	1.207	.107	.116	1.709	.873
6	.073	13.40	-1.561	1.272	.112	.130	1.883	1.036
7	.081	13.09	-1.701	1.325	.117	.144	2.056	1.194
8	.089	12.80	-1.835	1.369	.121	.158	2.228	1.349
9	.096	12.54	-1.964	1.406	.124	.171	2.400	1.502
10	.103	12.30	-2.089	1.438	.127	.184	2.570	1.653
20	.173	10.90	-3.237	1.618	.143	.308	4.261	3.113
40	.307	9.94	-5.360	1.745	.154	.545	7.601	5.945
60	.438	9.56	-7.426	1.796	.159	.779	10.924	8.746
80	.569	9.36	-9.475	1.824	.161	1.012	14.240	11.539
100	.700	9.24	-11.515	1.842	.163	1.244	17.554	14.327

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .180$ $\rho_\ell / \rho_v = 1000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.036	2.60	-.842	.850	.149	.072	1.199	.340
3	.048	3.10	-1.097	1.039	.182	.097	1.396	.557
4	.058	3.17	-1.296	1.157	.203	.117	1.590	.751
5	.068	3.14	-1.470	1.242	.218	.135	1.783	.934
6	.076	3.08	-1.629	1.307	.229	.153	1.975	1.111
7	.085	3.02	-1.779	1.360	.238	.170	2.166	1.282
8	.093	2.96	-1.922	1.403	.246	.186	2.356	1.451
9	.101	2.91	-2.060	1.440	.252	.202	2.545	1.618
10	.109	2.86	-2.195	1.471	.257	.218	2.734	1.782
20	.185	2.57	-3.440	1.646	.288	.369	4.602	3.377
40	.330	2.36	-5.762	1.766	.309	.661	8.299	6.482
60	.474	2.28	-8.030	1.815	.317	.948	11.979	9.560
80	.617	2.24	-10.280	1.841	.321	1.235	15.654	12.629
100	.760	2.22	-12.524	1.857	.324	1.521	19.326	15.694

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .180$ $\qquad\qquad$ $\rho_\ell / \rho_v = 2000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.036	5.19	-.842	.850	.105	.072	1.199	.340
3	.048	6.20	-1.096	1.038	.129	.096	1.395	.557
4	.058	6.34	-1.295	1.157	.144	.117	1.590	.751
5	.068	6.28	-1.469	1.241	.154	.135	1.782	.934
6	.076	6.16	-1.628	1.307	.162	.153	1.974	1.110
7	.085	6.03	-1.778	1.359	.169	.170	2.165	1.281
8	.093	5.91	-1.921	1.403	.174	.186	2.354	1.450
9	.101	5.80	-2.059	1.439	.179	.202	2.543	1.616
10	.109	5.71	-2.194	1.471	.182	.218	2.732	1.780
20	.184	5.12	-3.437	1.646	.204	.369	4.597	3.373
40	.330	4.71	-5.754	1.766	.219	.660	8.287	6.472
60	.473	4.55	-8.018	1.814	.225	.947	11.960	9.544
80	.616	4.47	-10.264	1.841	.228	1.233	15.627	12.606
100	.759	4.42	-12.502	1.857	.230	1.518	19.291	15.665

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .180 ρ_ℓ / ρ_v = 3000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.036	7.78	-.842	.850	.086	.072	1.199	.340
3	.048	9.29	-1.096	1.038	.105	.096	1.395	.557
4	.058	9.51	-1.295	1.156	.117	.117	1.589	.751
5	.068	9.41	-1.469	1.241	.126	.135	1.782	.933
6	.076	9.23	-1.628	1.307	.132	.153	1.974	1.110
7	.085	9.04	-1.778	1.359	.138	.170	2.164	1.281
8	.093	8.87	-1.921	1.403	.142	.186	2.354	1.450
9	.101	8.70	-2.059	1.439	.146	.202	2.543	1.616
10	.109	8.55	-2.193	1.471	.149	.218	2.731	1.780
20	.184	7.67	-3.436	1.646	.167	.369	4.596	3.372
40	.330	7.06	-5.752	1.766	.179	.659	8.284	6.470
60	.473	6.83	-8.014	1.814	.184	.946	11.955	9.540
80	.616	6.70	-10.260	1.841	.186	1.232	15.620	12.601
100	.759	6.62	-12.497	1.857	.188	1.518	19.283	15.658

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .180$ $\rho_\ell / \rho_v = 4000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.036	10.37	-.842	.850	.075	.072	1.199	.340
3	.048	12.39	-1.096	1.038	.091	.096	1.395	.557
4	.058	12.67	-1.295	1.156	.102	.117	1.589	.751
5	.068	12.54	-1.469	1.241	.109	.135	1.782	.933
6	.076	12.31	-1.628	1.307	.115	.153	1.974	1.109
7	.085	12.06	-1.778	1.359	.119	.170	2.164	1.281
8	.093	11.82	-1.921	1.403	.123	.186	2.354	1.449
9	.101	11.60	-2.059	1.439	.126	.202	2.543	1.616
10	.109	11.40	-2.193	1.471	.129	.218	2.731	1.780
20	.184	10.23	-3.436	1.646	.144	.369	4.595	3.371
40	.330	9.41	-5.752	1.766	.155	.659	8.282	6.468
60	.473	9.10	-8.013	1.814	.159	.946	11.953	9.538
80	.616	8.93	-10.258	1.841	.162	1.232	15.617	12.598
100	.759	8.82	-12.495	1.857	.163	1.517	19.279	15.655

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .180$ $\rho_\ell / \rho_v = 5000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.036	12.97	-.842	.850	.067	.072	1.199	.340
3	.048	15.48	-1.096	1.038	.082	.096	1.395	.556
4	.058	15.84	-1.295	1.156	.091	.117	1.589	.751
5	.068	15.68	-1.469	1.241	.098	.135	1.782	.933
6	.076	15.38	-1.628	1.307	.103	.153	1.974	1.109
7	.085	15.07	-1.778	1.359	.107	.170	2.164	1.281
8	.093	14.77	-1.921	1.403	.110	.186	2.354	1.449
9	.101	14.50	-2.059	1.439	.113	.202	2.543	1.615
10	.109	14.25	-2.193	1.471	.116	.218	2.731	1.780
20	.184	12.78	-3.435	1.646	.129	.369	4.595	3.371
40	.330	11.77	-5.751	1.766	.139	.659	8.282	6.468
60	.473	11.37	-8.013	1.814	.142	.946	11.952	9.537
80	.616	11.16	-10.257	1.841	.145	1.232	15.616	12.597
100	.759	11.03	-12.493	1.857	.146	1.517	19.277	15.653

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .200

ρ_ℓ / ρ_v = 1000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.037	2.88	-.866	.876	.138	.082	1.217	.358
3	.050	3.46	-1.131	1.068	.169	.110	1.431	.588
4	.060	3.56	-1.340	1.187	.187	.134	1.643	.795
5	.070	3.53	-1.522	1.272	.201	.156	1.853	.991
6	.079	3.48	-1.691	1.338	.211	.176	2.062	1.179
7	.088	3.41	-1.849	1.390	.219	.196	2.269	1.364
8	.097	3.35	-2.001	1.433	.226	.216	2.476	1.545
9	.106	3.30	-2.149	1.469	.232	.235	2.682	1.723
10	.114	3.25	-2.292	1.500	.236	.253	2.888	1.900
20	.195	2.95	-3.626	1.670	.263	.433	4.923	3.619
40	.352	2.74	-6.131	1.785	.281	.782	8.953	6.975
60	.507	2.65	-8.583	1.831	.288	1.126	12.967	10.305
80	.661	2.61	-11.019	1.855	.292	1.469	16.975	13.626
100	.815	2.58	-13.448	1.871	.294	1.812	20.981	16.944

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .200$ $\rho_\ell / \rho_v = 2000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.037	5.76	-.866	.876	.098	.082	1.217	.358
3	.050	6.92	-1.131	1.068	.119	.110	1.431	.588
4	.060	7.11	-1.339	1.187	.133	.134	1.642	.795
5	.070	7.06	-1.522	1.272	.142	.156	1.852	.990
6	.079	6.95	-1.690	1.338	.149	.176	2.061	1.179
7	.088	6.82	-1.849	1.390	.155	.196	2.269	1.363
8	.097	6.70	-2.001	1.433	.160	.215	2.475	1.543
9	.106	6.59	-2.148	1.469	.164	.234	2.681	1.722
10	.114	6.49	-2.291	1.500	.167	.253	2.886	1.899
20	.195	5.88	-3.624	1.670	.186	.433	4.919	3.615
40	.351	5.46	-6.124	1.785	.199	.781	8.943	6.967
60	.506	5.30	-8.573	1.830	.204	1.124	12.950	10.291
80	.660	5.21	-11.005	1.855	.207	1.467	16.952	13.607
100	.814	5.15	-13.430	1.871	.209	1.809	20.952	16.920

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg}$ = .200

ρ_ℓ / ρ_v = 3000

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.037	8.64	-.866	.876	.080	.082	1.217	.358
3	.050	10.37	-1.131	1.068	.097	.110	1.431	.588
4	.060	10.66	-1.339	1.187	.108	.134	1.642	.795
5	.070	10.58	-1.522	1.272	.116	.156	1.852	.990
6	.079	10.42	-1.690	1.337	.122	.176	2.061	1.178
7	.088	10.23	-1.848	1.390	.127	.196	2.268	1.362
8	.097	10.05	-2.000	1.433	.131	.215	2.475	1.543
9	.106	9.88	-2.147	1.469	.134	.234	2.681	1.722
10	.114	9.73	-2.291	1.500	.137	.253	2.886	1.898
20	.195	8.82	-3.623	1.670	.152	.433	4.918	3.615
40	.351	8.18	-6.123	1.785	.163	.780	8.940	6.964
60	.506	7.94	-8.570	1.830	.167	1.124	12.946	10.288
80	.660	7.81	-11.002	1.855	.169	1.466	16.947	13.603
100	.814	7.72	-13.426	1.871	.171	1.808	20.945	16.914

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .200$ $\rho_\ell / \rho_v = 4000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	- 0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.037	11.52	-.866	.876	.069	.082	1.217	.358
3	.050	13.83	-1.131	1.068	.084	.110	1.431	.588
4	.060	14.21	-1.339	1.187	.094	.134	1.642	.795
5	.070	14.11	-1.522	1.272	.101	.156	1.852	.990
6	.079	13.89	-1.690	1.337	.106	.176	2.061	1.178
7	.088	13.64	-1.848	1.390	.110	.196	2.268	1.362
8	.097	13.39	-2.000	1.433	.113	.215	2.475	1.543
9	.105	13.17	-2.147	1.469	.116	.234	2.681	1.722
10	.114	12.97	-2.291	1.500	.119	.253	2.886	1.898
20	.195	11.76	-3.623	1.670	.132	.433	4.918	3.614
40	.351	10.91	-6.122	1.785	.141	.780	8.939	6.964
60	.506	10.58	-8.569	1.830	.145	1.124	12.945	10.286
80	.660	10.41	-11.000	1.855	.147	1.466	16.944	13.601
100	.814	10.30	-13.424	1.871	.148	1.808	20.941	16.912

TABULATION OF SOLUTIONS

$C_p \Delta T_s / h_{fg} = .200$ $\rho_\ell / \rho_v = 5000$

u_v/u_ℓ	S	f	ϕ_ℓ	$\Delta\phi_\ell$	$\Delta\phi_v$	$F(\phi_i)$	$F'(\phi_i)$	$F''(\phi_i)$
1	0.000	0.00	0.000	0.000	0.000	0.000	1.000	0.000
2	.037	14.40	-.866	.876	.062	.082	1.217	.358
3	.050	17.29	-1.131	1.068	.075	.110	1.431	.588
4	.060	17.76	-1.339	1.187	.084	.134	1.642	.795
5	.070	17.64	-1.522	1.272	.090	.156	1.852	.990
6	.079	17.36	-1.690	1.337	.095	.176	2.061	1.178
7	.088	17.04	-1.848	1.390	.098	.196	2.268	1.362
8	.097	16.74	-2.000	1.433	.101	.215	2.475	1.543
9	.105	16.46	-2.147	1.469	.104	.234	2.680	1.721
10	.114	16.21	-2.290	1.500	.106	.253	2.886	1.898
20	.195	14.69	-3.623	1.670	.118	.433	4.917	3.614
40	.351	13.64	-6.122	1.785	.126	.780	8.939	6.963
60	.506	13.22	-8.569	1.830	.129	1.124	12.944	10.286
80	.660	13.00	-11.000	1.855	.131	1.466	16.943	13.600
100	.814	12.87	-13.423	1.871	.132	1.808	20.940	16.910

2. Discussion of Solutions

This work represents the first attempt at an analytical modeling of this system. Some of the assumptions made may be debatable. Great care should be taken in ascertaining that $\frac{T_v - T_i}{T_v - T_\ell}$ is indeed negligible. The assumption that Prandtl's old theory of turbulence holds is at best an approximation. The boundary condition $v(\phi_\ell) = 0$ is unverified. The solutions must be compared to experimental data. No experimental data obtained from a system similar to the one analyzed could be found in the literature.

Many measurements have been published for film condensation processes. However, these data have only limited applicability to our system, since film processes should exhibit much lower heat transfer rates. This is due to the presence of a wall which limits turbulent behavior.

J. H. Linehan [7] found the Stanton Number to be a constant (0.00753) in his experiments with condensation on a thin turbulent film flowing past an adiabatic wall. However, his experiments cover only a small range of parameters. When his data are compared to our solution, it is found that, while our theory predicts higher Stanton Numbers (as it should), it too predicts them to be roughly constant over the limited range of temperatures and velocities studied by Linehan.

E. K. Levy [6] performed tests on an axially symmetric system. His vapor flow was supersonic and his boundary layer is bounded by the geometry of his apparatus. His Stanton Numbers are again lower than those predicted by this theory.

Abramovich [1] analyzed a system in which the free vapor was stationary, a condition which this model can not approach.

While it is not expected that these solutions can be closely correlated to experimental data, it is anticipated that the trends hold and that this theory will provide an order-of-magnitude indication.

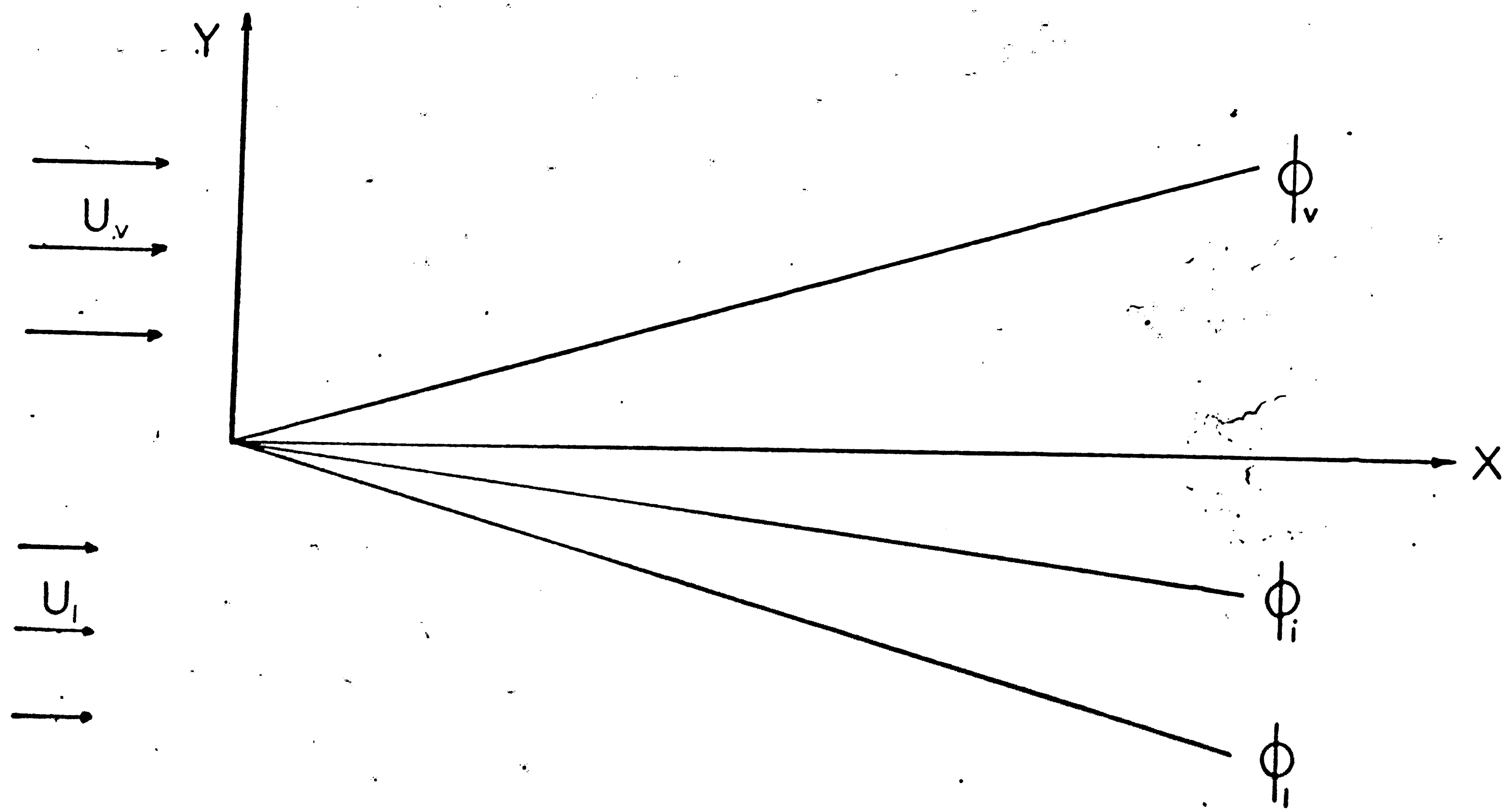
As exhibited in Figures 7 through 9, the Stanton Number increases linearly with the velocity ratio (for $u_v/u_\ell > 5$), increases more rapidly with $C_p \Delta T_s / h_{fg}$, and is nearly independent of the density ratio.

The behavior of the interfacial friction factor is shown on Figures 10 through 12. It increases linearly with the density ratio, increases faster with $C_p \Delta T_s / h_{fg}$ and follows the expected trend with the velocity ratio. That is, at $u_v = u_\ell$, the shear stress should be zero and it should soon after assume its maximum value since friction factors are reported to be highest at low Reynolds numbers.

APPENDIX A

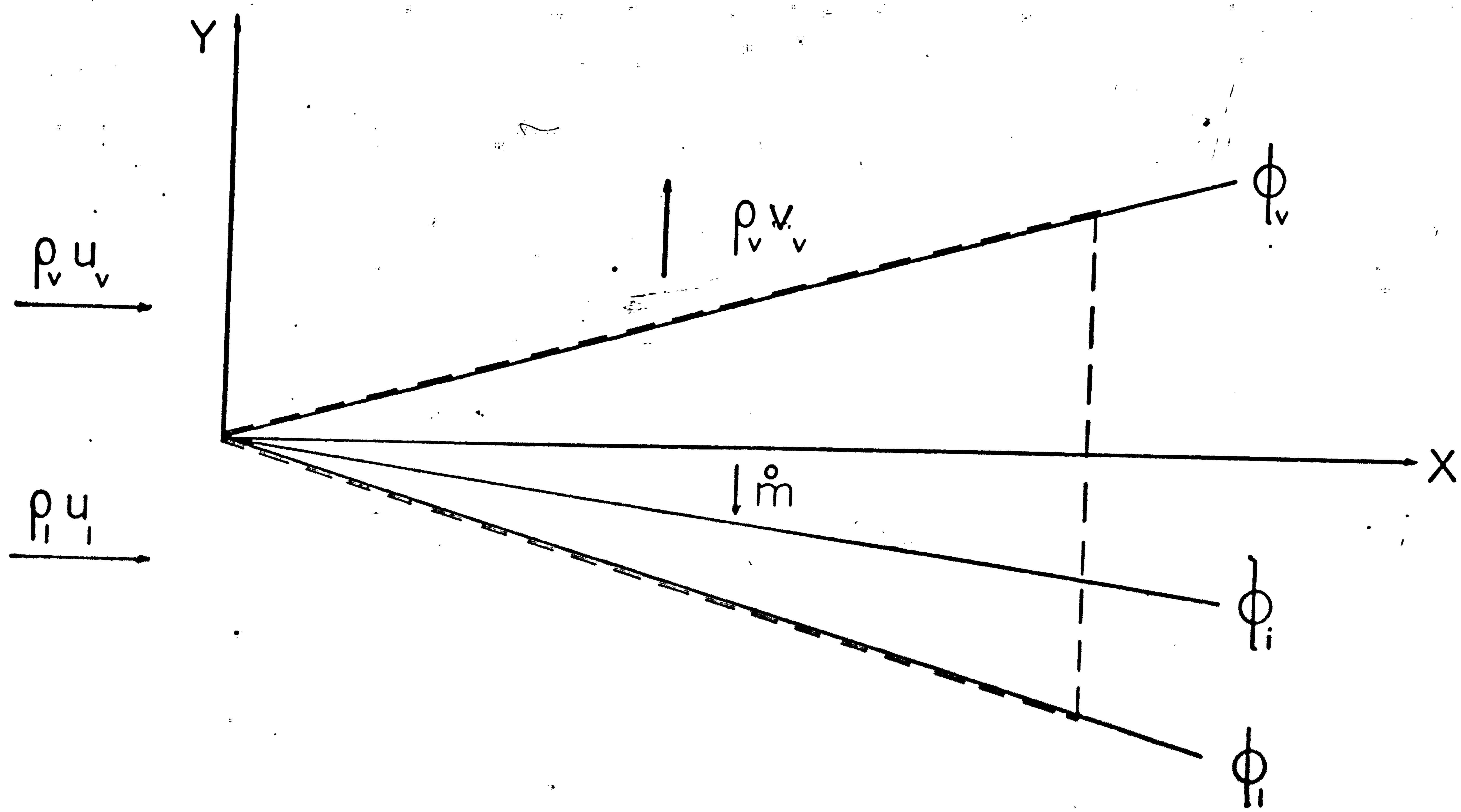
FIGURES

FIGURE 1



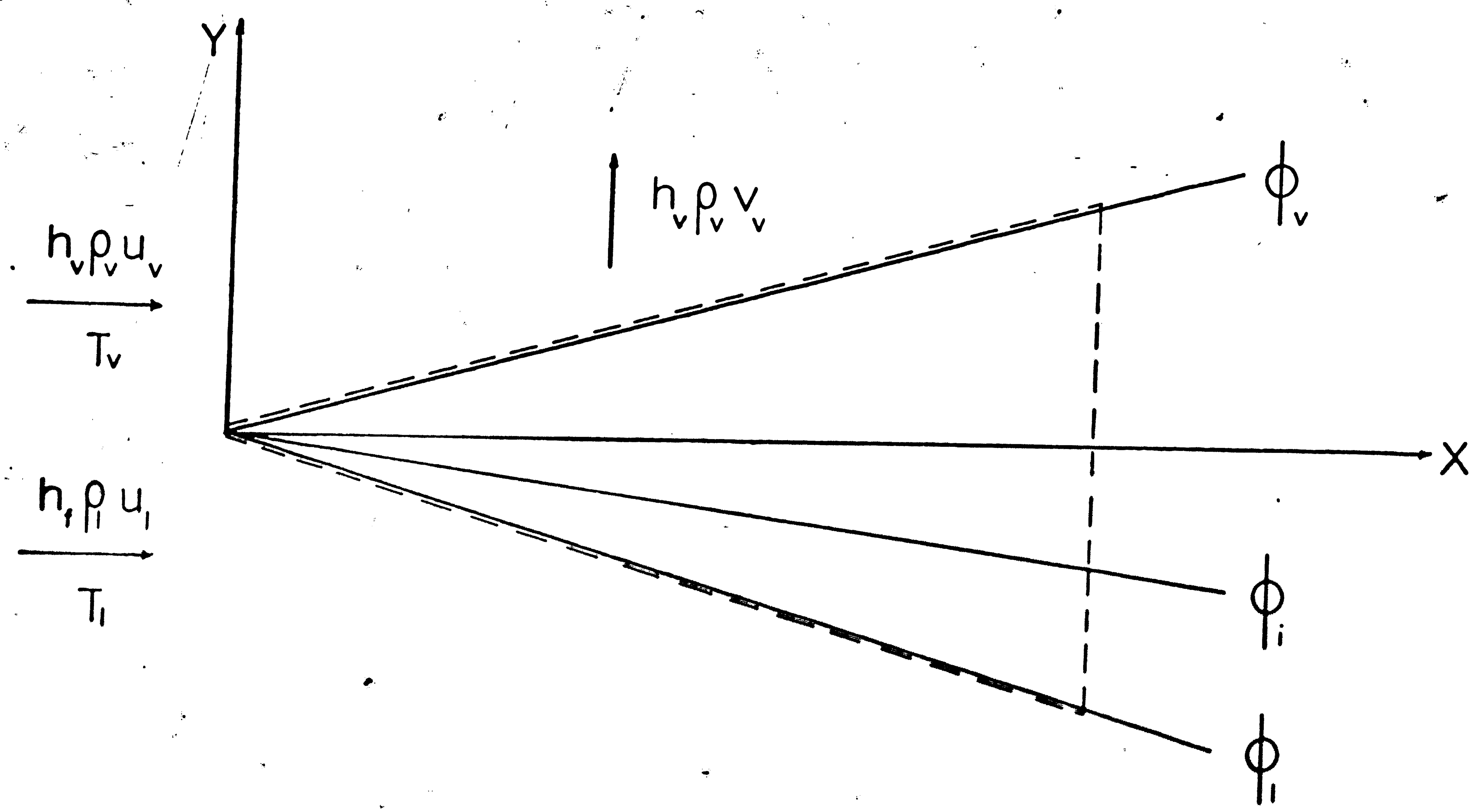
SCHEMATIC FOR SYSTEM

FIGURE 2



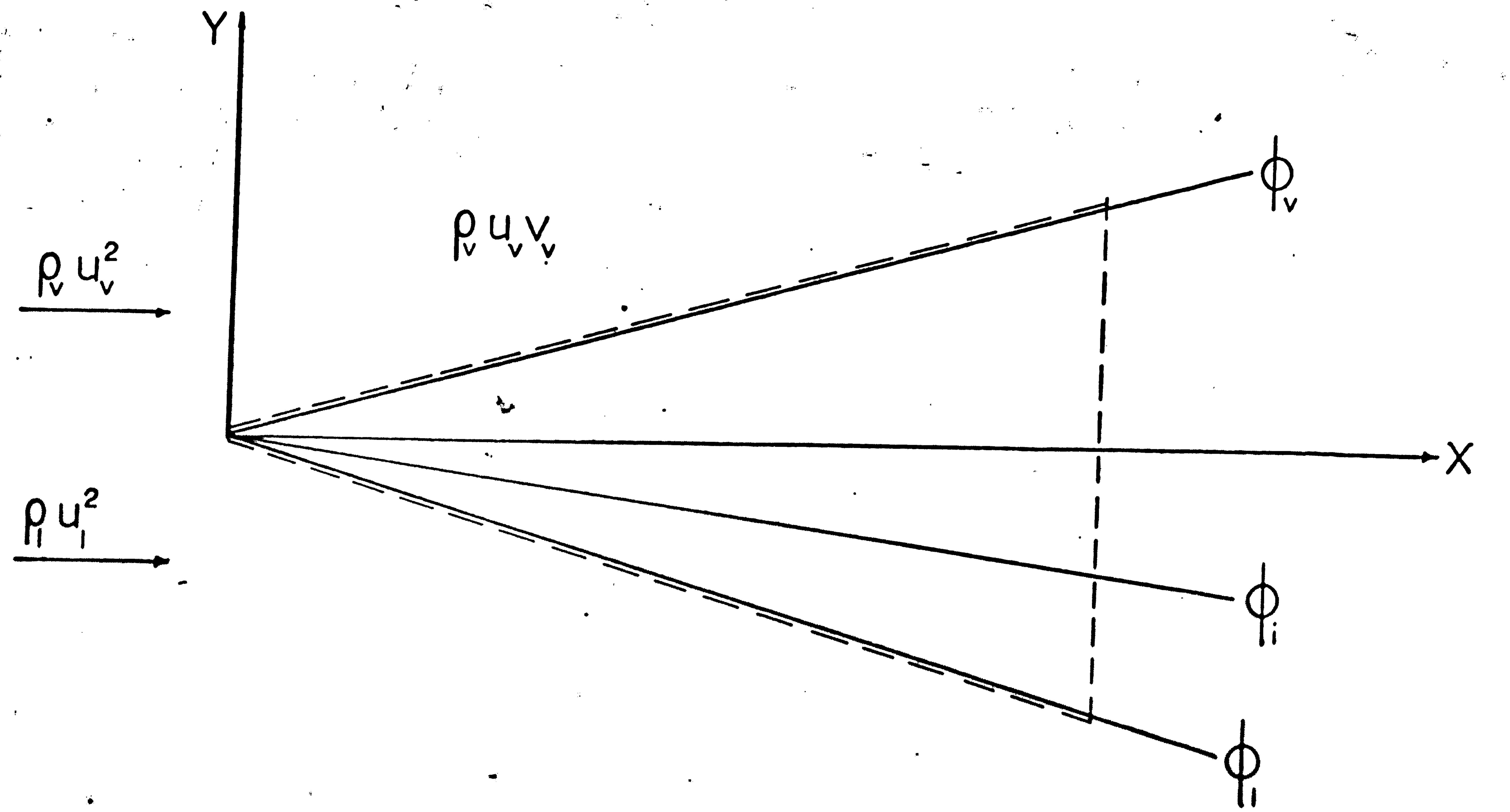
CONTROL VOLUME FOR MASS BALANCE

FIGURE 3



CONTROL VOLUME FOR ENERGY BALANCE

FIGURE 4



CONTROL VOLUME FOR MOMENTUM BALANCE

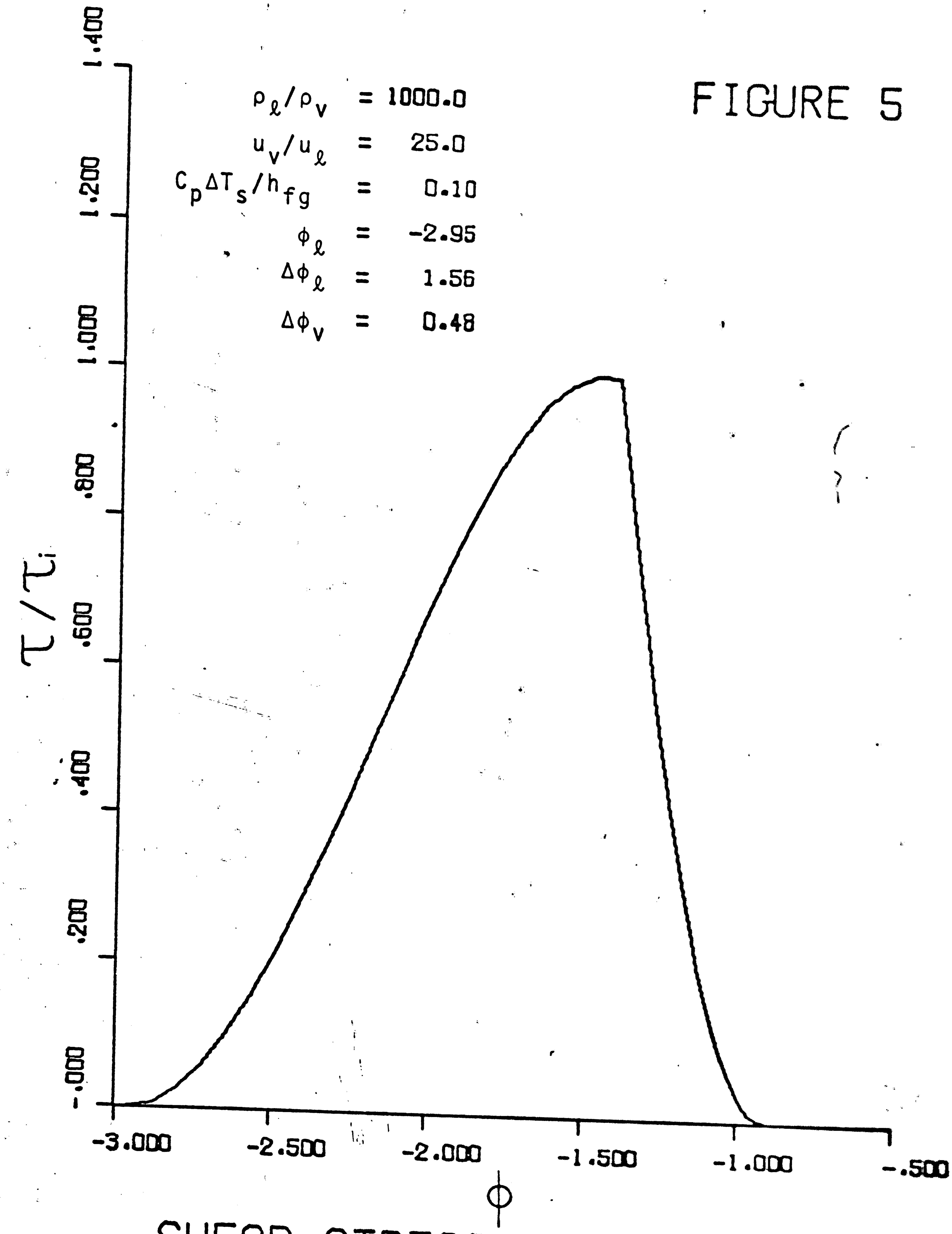
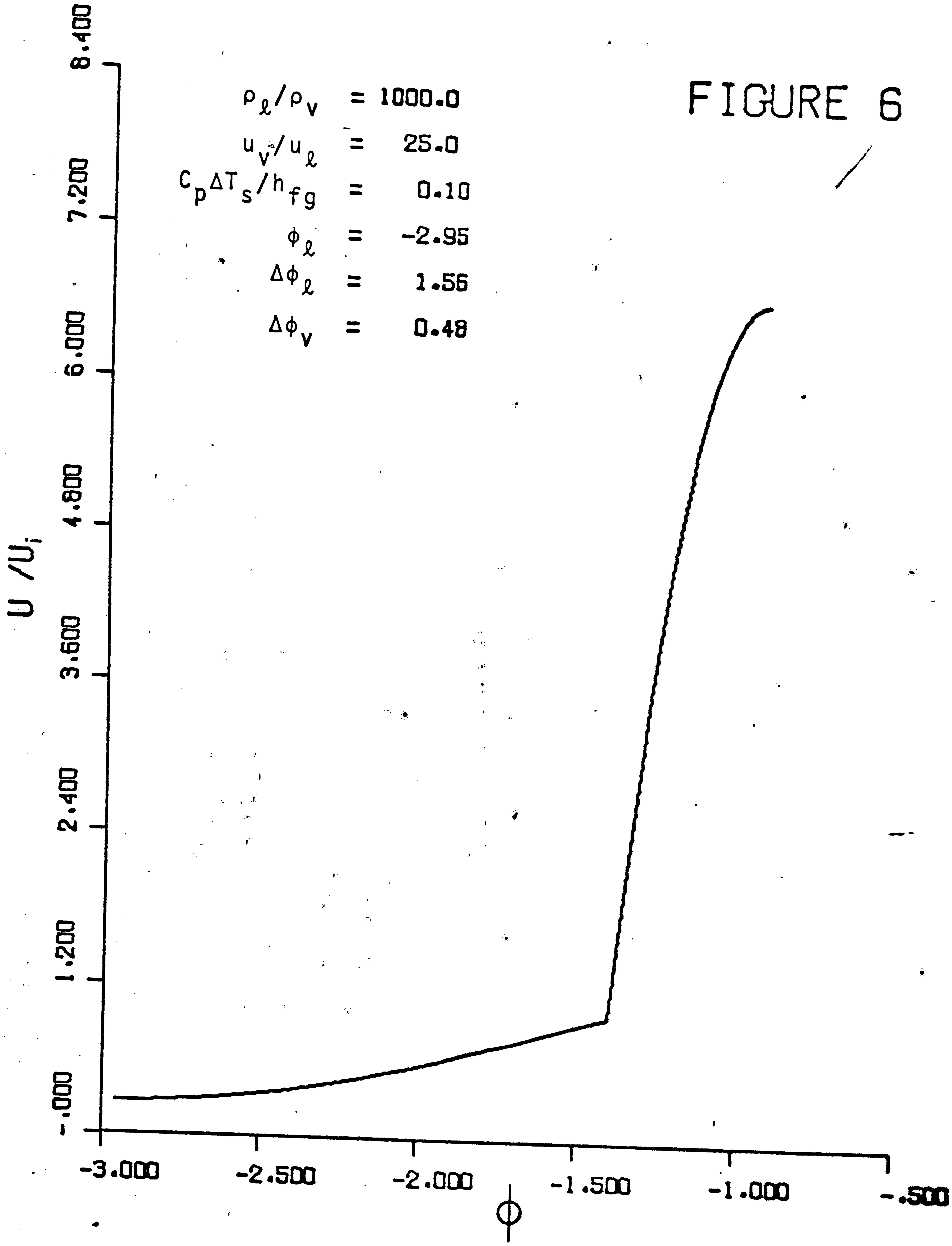
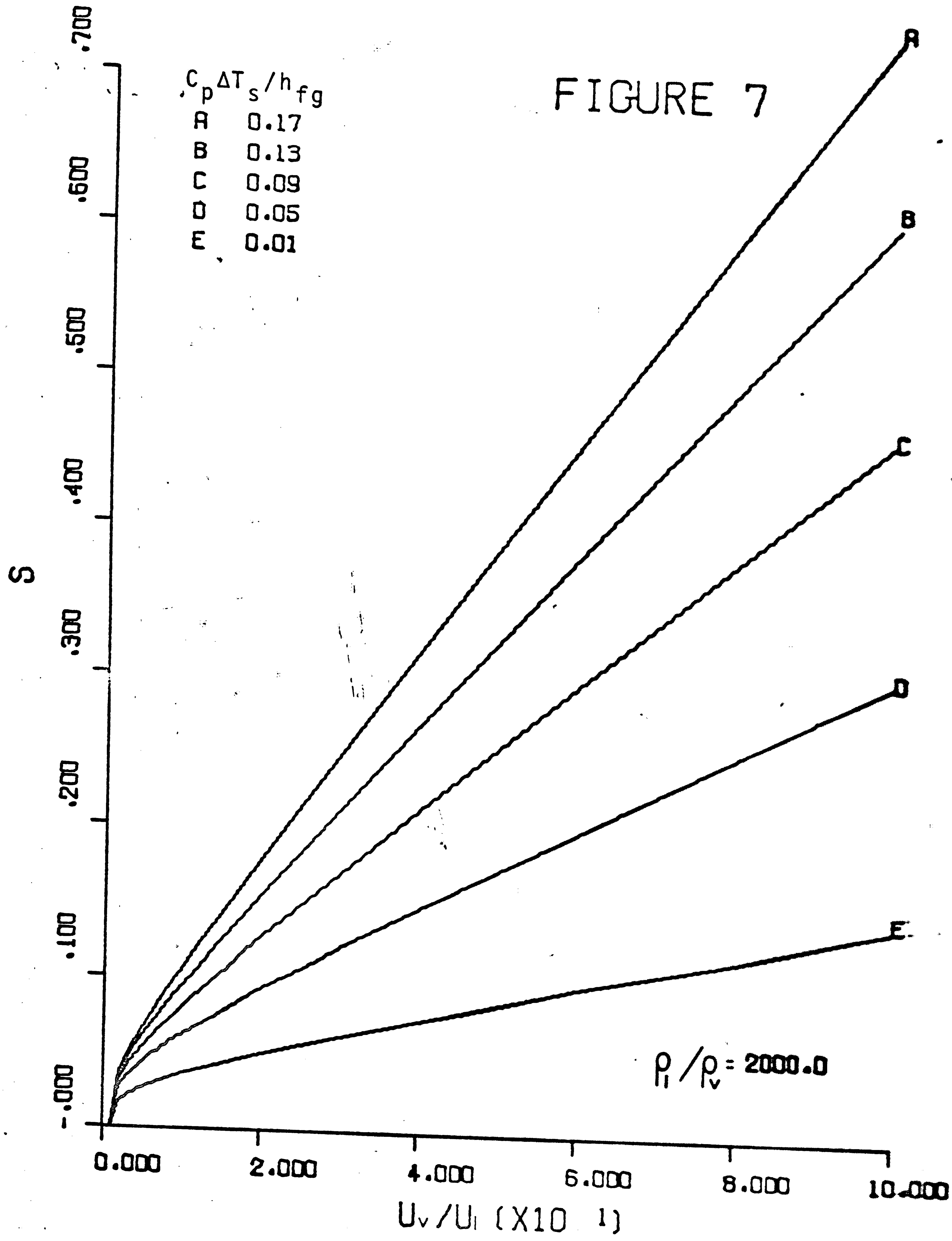


FIGURE 5

SHEAR STRESS DISTRIBUTION



HORIZONTAL VELOCITY DISTRIBUTION



STANTON NUMBER VERSUS U_v / U_l

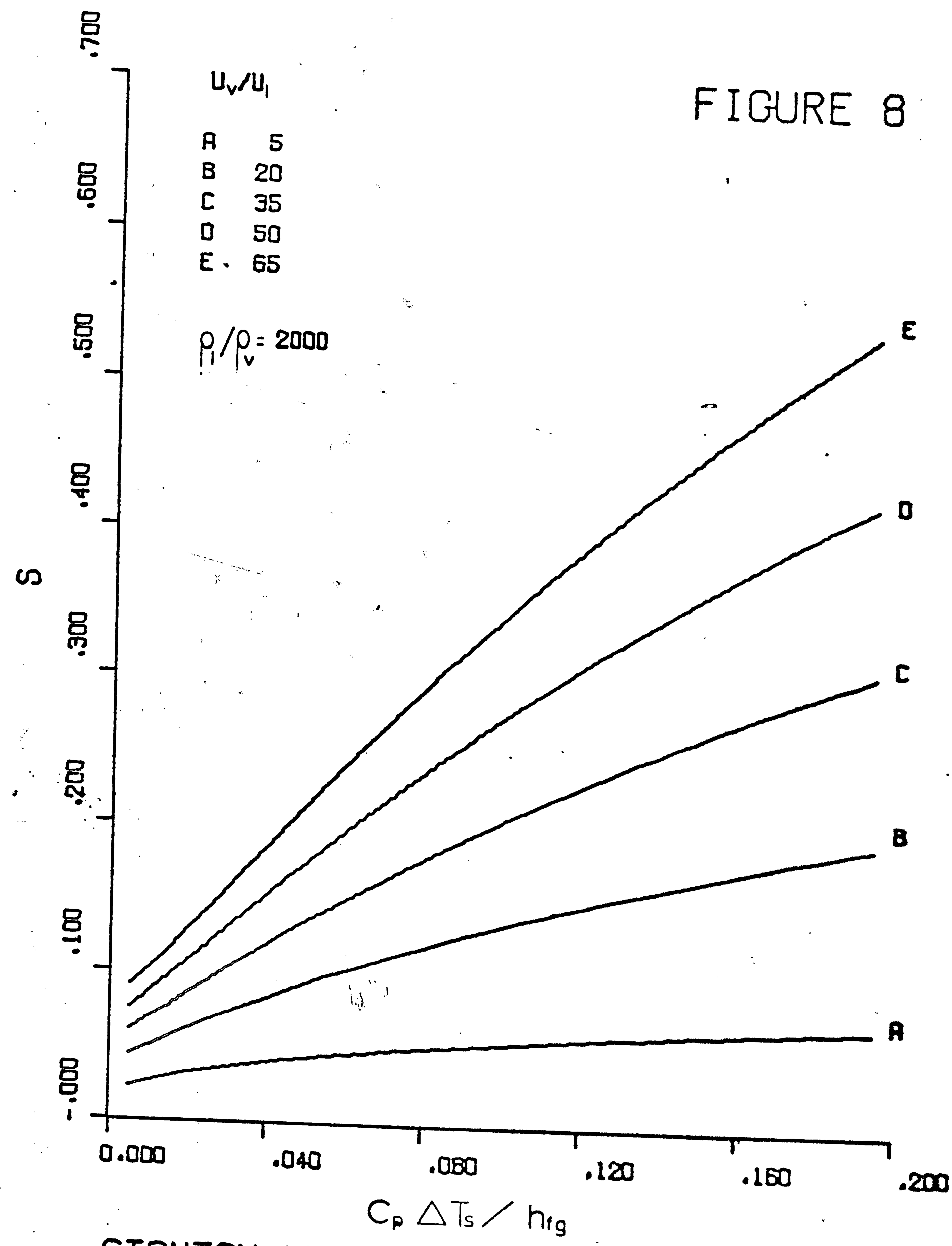
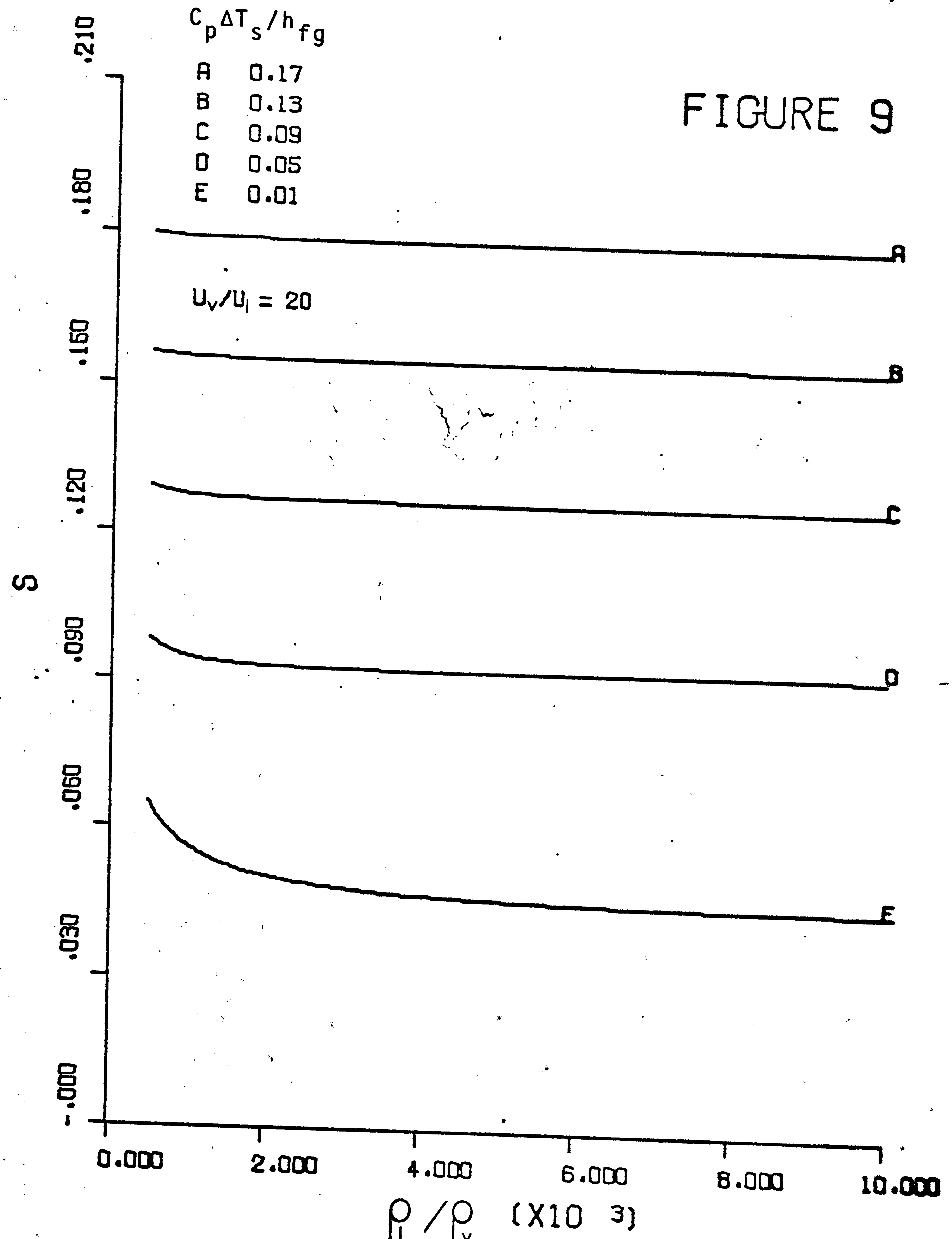
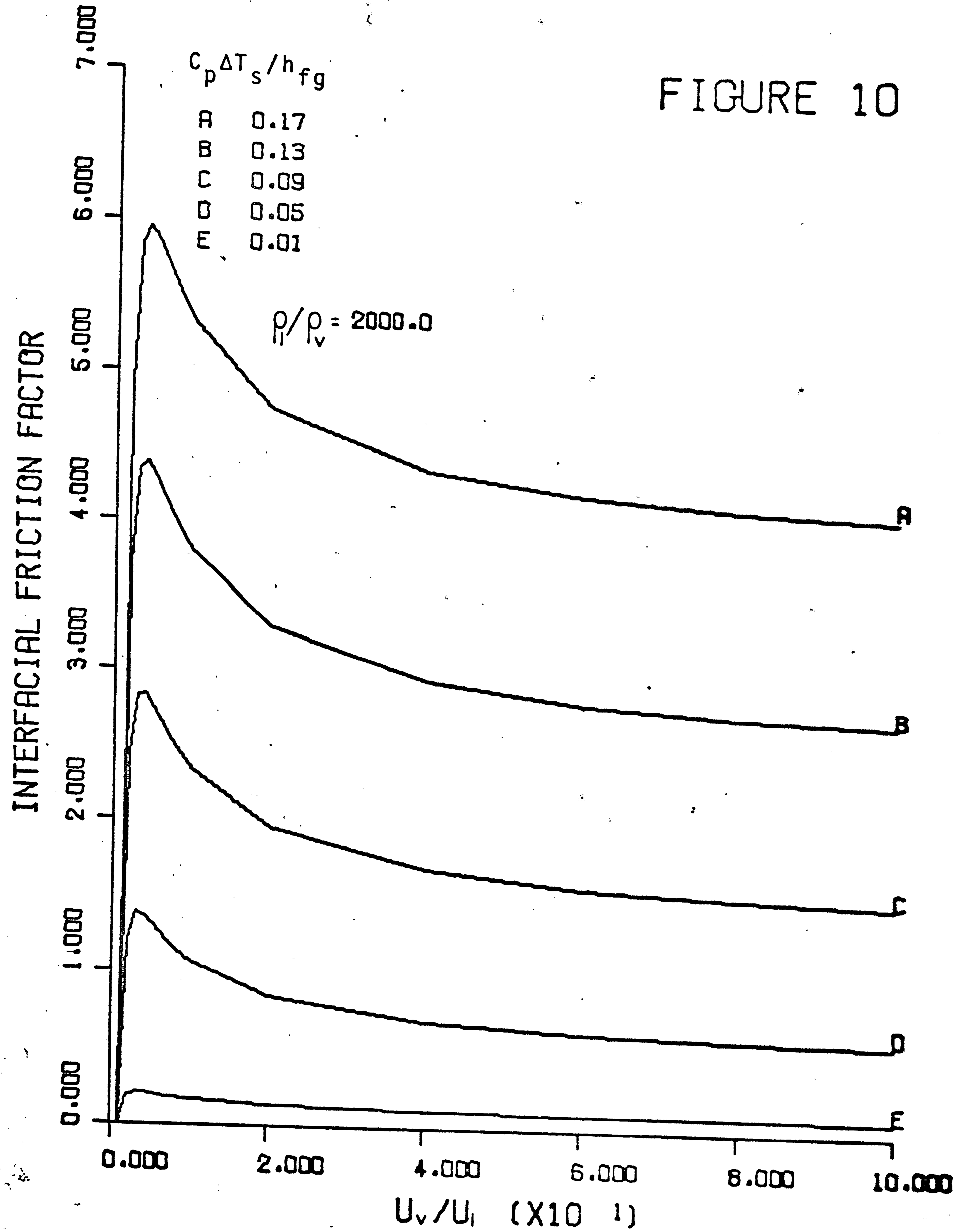


FIGURE 8

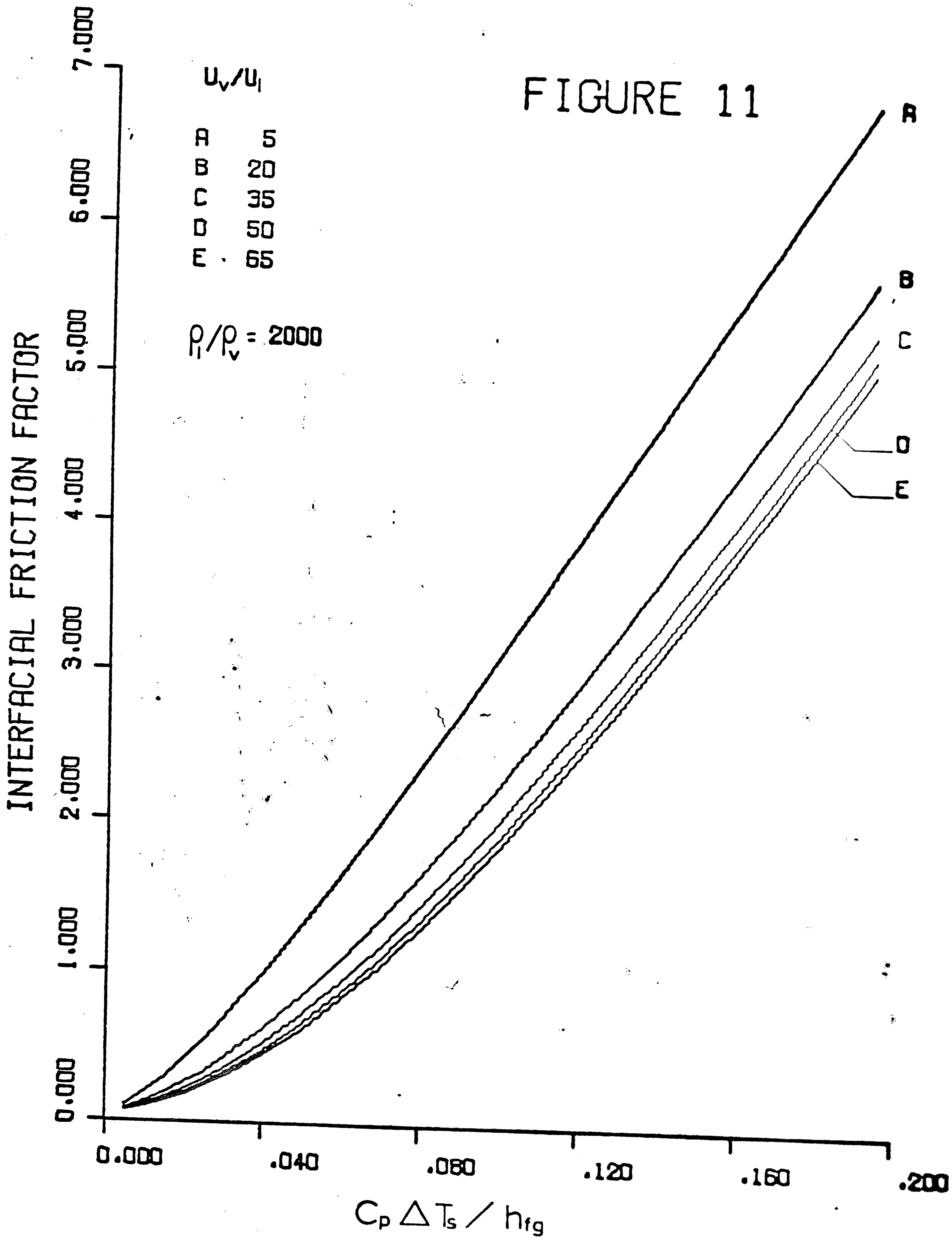
STANTON NUMBER VERSUS CP(TV-TL)/HFG



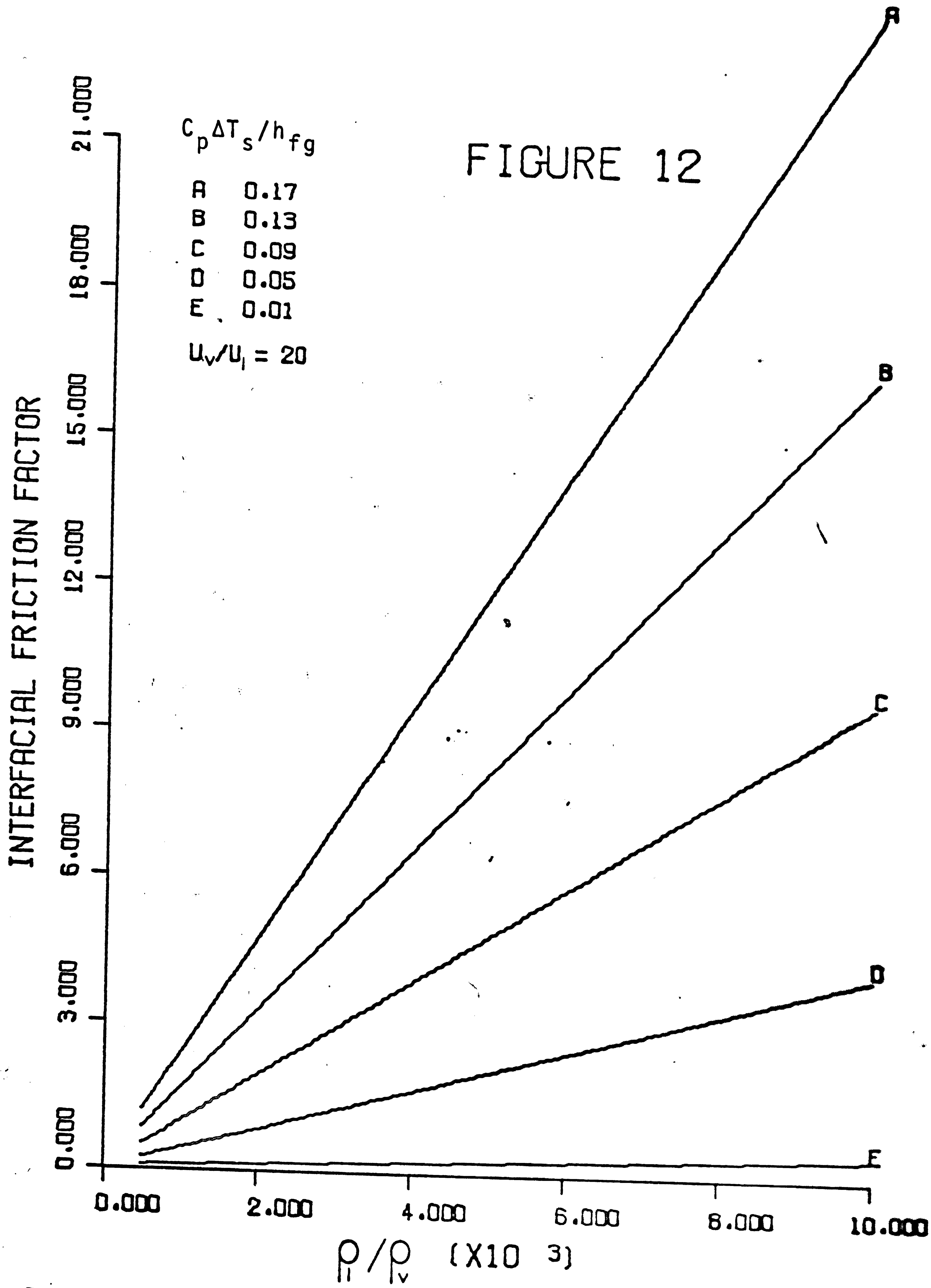
STANTON NUMBER VERSUS DENSITY RATIO



INTERFACIAL FRICTION FACTOR VERSUS U_v / U_l



INTERFACIAL FRICTION FACTOR VERSUS CP(TV-TL)/HFG



INTERFACIAL FRICTION FACTOR VERSUS DENSITY RATIO

APPENDIX B

NOMENCLATURE

SYMBOLS

a	an empirical constant (0.09)
A_1, A_2, A_3	constants of integration in equation (17)
b	constant in equation (4)
C_1, C_2, C_3	constants of integration in equation (20)
C_p	constant pressure specific heat of the liquid
d	constant defined by equation (38)
f	interfacial friction factor
g	a function, defined by equation (3)
$F(\phi)$	flow function of the liquid
$F'(\phi)$	$dF/d\phi$
$F''(\phi)$	$d^2F/d\phi^2$
$G(\phi)$	flow function of the vapor
$G'(\phi)$	$dG/d\phi$
$G''(\phi)$	$d^2G/d\phi^2$
h	convection heat transfer coefficient
h_o	enthalpy of liquid at reference temperature T_o
h_f	enthalpy of liquid
h_{fg}	latent heat of condensation
h_v	enthalpy of the vapor
L	mixing length
$\dot{m}$	condensation rate per unit area
q/A	heat transfer rate per unit area
r	a function, defined by equation (7)
S	Stanton Number of liquid
T	temperature

u	x-component of velocity
v	y-component of velocity
x, y	rectangular coordinates
α	$\phi - \phi_\ell$
β	$\phi_v - \phi$
γ	$\phi - \phi_2$
$\Delta\phi_\ell$	dimensionless liquid boundary layer thickness
$\Delta\phi_v$	dimensionless vapor boundary layer thickness
ΔT_s	$T_v - T_\ell$
τ	shear stress acting on fluid
ϕ	dimensionless coordinate (y/ax)
ψ	stream function

SUBSCRIPTS

i	at the interface
ℓ	at the liquid boundary layer edge
v	at the vapor boundary layer edge

APPENDIX C

METHOD OF COMPUTATION

The computations were performed on a digital computer. The scheme employed was the Newton-Raphson method for two unknowns. That is, given two functions

$$F_1(x,y) = 0$$

$$F_2(x,y) = 0$$

then the first order approximation of these functions can be obtained from the first few terms of their Taylor Series expansions:

$$F_1(x,y) = F_1\Big|_{x_o,y_o} + \frac{\partial F_1}{\partial x}\Big|_{x_o,y_o} (x-x_o)$$

$$+ \frac{\partial F_1}{\partial y}\Big|_{x_o,y_o} (y-y_o) = 0$$

$$F_2(x,y) = F_2\Big|_{x_o,y_o} + \frac{\partial F_2}{\partial x}\Big|_{x_o,y_o} (x-x_o)$$

$$+ \frac{\partial F_2}{x_o,y_o} (y-y_o) = 0$$

where x_o and y_o are prior approximations for x and y. Solving for x and y:

$$x = x_o + \frac{1}{D} \left[\frac{\partial F_1}{\partial y} \cdot F_2 - \frac{\partial F_2}{\partial y} \cdot F_1\right]\Big|_{x_o,y_o} \qquad \text{(C1)}$$

$$y = y_o + \frac{1}{D}\left[\frac{\partial F_2}{\partial x} \cdot F_1 - \frac{\partial F_1}{\partial x} \cdot F_2\right]\Bigg|_{x_o,y_o} \qquad \text{(C2)}$$

where

$$D = \left[\frac{\partial F_1}{\partial x}\frac{\partial F_2}{\partial y} - \frac{\partial F_1}{\partial y}\frac{\partial F_2}{\partial x}\right]\Bigg|_{x_o,y_o} .$$

Equations (C1) and (C2) are solved simultaneously until x and y approach x_o and y_o within some small limit.

APPENDIX D

THE INTERFACIAL TEMPERATURE DISCONTINUITY

Theories exist which permit us to express the interfacial temperature drop in terms of physical parameters. The most widely used theory is that of Schrage [12]:

$$\frac{q}{A} = \frac{(T_v - T_i)\sigma P h_{fg}^2}{(1 - 0.563)\sigma \; (2\pi R^3)^{1/2}(T_v)^{5/2}} \quad . \tag{D1}$$

Where q/A is the heat transfer rate per unit area due to condensation, P is the pressure of the system, R the gas constant of the vapor and σ the condensation coefficient. And since

$$\frac{q}{A} = \dot{m} h_{fg} \,, \tag{D2}$$

$$\frac{T_v - T_i}{T_v - T_\ell} = \frac{a(1 - 0.523\sigma) T_v^{5/2} (2\pi R^3)^{1/2} \rho_\ell u_\ell F(\phi_i)}{\sigma \Delta T_s P h_{fg}} \tag{D3}$$

with the aid of equation (27).

Depending on the fluid properties and the flow parameters, the range of magnitudes of the temperature ratio is

$$0 < \frac{T_v - T_i}{T_v - T_\ell} < 1 \quad . \tag{D4}$$

While some arguments exist concerning the magnitude of σ, most workers in this field have settled on σ = 1.0 (see [9]).

Equation (53) predicts $\frac{T_v - T_i}{T_v - T_\ell}$ to be smallest when $\frac{T_v^{5/2}}{P}$ and ΔT_s are small (since $\frac{F(\phi_i)}{\Delta T_s}$ increases with increasing ΔT_s).

To get a notion concerning the magnitude and variation of the temperature ratio, consider the following example: Water at

$$T_v = 212°F \quad , \quad u_v = 300 \text{ ft/sec}$$

$$T_\ell = 115°F \quad , \quad u_\ell = 15 \text{ ft/sec}$$

then

$$\frac{u_v}{u_\ell} = 20 \qquad \frac{C_p \Delta T_s}{h_{fg}} = 0.1 \qquad \frac{\rho_\ell}{\rho_v} = 1720$$

$$\therefore \frac{T_v - T_i}{T_v - T_\ell}$$

$$= \frac{0.09 * 0.477(672)^{5/2}(6.28)^{1/2}(85.8)^{3/2} 60 * 15 * 0.15}{97 * 14.7 * 970.3 * 144 * 778 * (32.17)^{1/2}}$$

$$= 0.15$$

This value represents roughly an upper bound to the assumption that $\frac{T_v - T_i}{T_v - T_\ell}$ is negligible in equation (32).

So we expect the assumption to hold when P is larger and/or ΔT_s smaller.

Consider some more examples, all of them for water and:

u_v = 300 ft/sec $\qquad$ $C_p \Delta T_s / h_{fg} = 0.1$

u_ℓ = 15 ft/sec $\qquad$ $u_v / u_\ell = 20$

Case 2)

T_v = 300°F $\qquad$ $\dfrac{T_v - T_i}{T_v - T_\ell} = 0.0495$

T_ℓ = 209°F

Case 3)

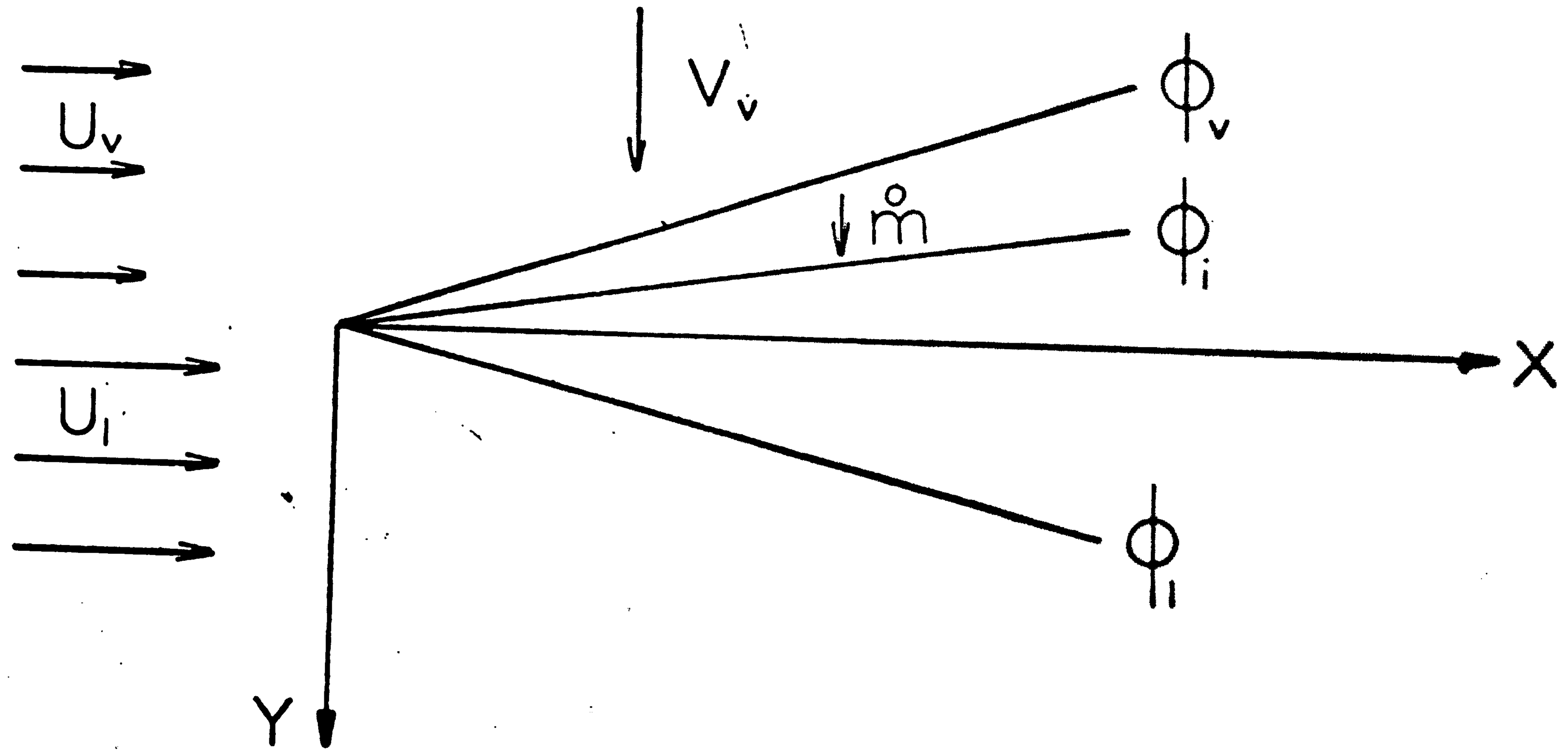
T_v = 180°F $\qquad$ $\dfrac{T_v - T_i}{T_v - T_\ell} = 0.24$

T_ℓ = 81°F

It follows that values read from our tables apply for high pressures and moderate ΔT_s. At high ΔT_s and low pressures recalculation of equations (32), (36) and (37) is required.

APPENDIX E

SOLUTION FOR THE CASE $u_v < u_\ell$



This thesis has so far only treated the case $u_v > u_\ell$. However, by rotating the y axis as indicated above, the solution can be easily extended to the case $u_v < u_\ell$. It can be shown that all equations up to (61), with the exceptions of equations (26), (27), (28), (54) and (55), hold.

The excepted equations will here be given in their revised form so as to apply for the case under discussion.

$$\dot{m}x = -[\int_{y_\ell}^{y_i} \rho_\ell u dy - \int_{y_\ell}^{0} u_\ell \rho_\ell dy]$$

(26a)

$$\dot{m}x = -[-\int_{y_i}^{y_v} \rho_v u dy + \int_{0}^{y_v} \rho_v u_v dy - \int_{0}^{x} v\rho_v dx]$$

$$\dot{m} = - a\rho_\ell u_\ell F(\phi_i) \tag{27a}$$

$$\dot{m} = - a\rho_v u_v G(\phi_i) \tag{28a}$$

$$h = - \frac{a\rho_\ell u_\ell h_{fg} F(\phi_i)}{\Delta T_s} \tag{54a}$$

$$\dot{S} = - \frac{a h_{fg} F(\phi_i)}{C_p \Delta T_s} \tag{55a}$$

Furthermore, equations (62) and (D3) are changed by multiplying their right sides by -1.

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VITA

Heinz Jaster was born in Berlin, Germany, on August 23, 1938, the son of Otto and Alice Jaster.

He attended Das Polygraphische Institut, Berlin, graduating as a journeyman printer in August 1956. After working as a printer in Berlin, Hanover and Frankfurt he immigrated to the United States in 1959. He was employed as a printer in New York, Chicago and San Francisco until 1963, when he began two years of service in the United States Army.

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Heinz Jaster is married to the former Anne Frances MacDowell. They have two children.